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TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

L. Lusternik. On Categories of Some Arc Families 131

I. Shafarevich. On the Normalizability of Topological Fields 133

ASTRONOMIE

B. Fessenkoff, de l'Académie, et **O. P. Cramer.** Sur la polarisation de la lumière réfléchie par la Lune 136

HYDRAULIQUE

F. B. Nelson-Skorniakov. Filtration from Bilaterally Drained Channel of Rectangular Section 138

PHYSIQUE MATHÉMATIQUE

G. A. Grünberg. Relation between Operational Expressions of Two Arbitrary Functions and the Operational Representation of their Product 141

PHYSIQUE DU GLOBE

M. Shvez. On the Problem of Unstationary Distribution of Air Temperature Near the Underlying Surface 144

CHIMIE Pages

A. V. Babaeva. Absorption Spectra of Complex Compounds, Cis-trans-isomerism of Complex Platinum Compounds of the Diamine Series and Light Absorption . 148

A. A. Balandin and **V. V. Patrikeev.** Formation of Coal Dendrites by Catalytic Decomposition of Alcohols on Metals 152

N. A. Toropov and **P. F. Konovalov.** Solid Solutions of Calcium and Strontium Orthosilicates 155

CHIMIE BIOLOGIQUE

D. A. Bochvar and **M. M. Shemiakin.** On the Methods for Obtaining and on the Structure of Some Water Soluble Analogues of Vitamin K 158

I. A. Smorodintzev and **N. V. Nikolaeva.** Variation of Activity of Tryptase upon Autolysis of Muscular Tissue 161

OCÉANOLOGIE

V. G. Bogorov. Diurnal Vertical Migration of Zooplankton in Arctic Seas 164

PHYTOPATHOLOGIE

K. S. Sukhov. Proof of Infectiousness of a Purified Protein Preparation of Oat Mosaic Virus (Zakuklivanie in Oats) 167

ENTOMOLOGIE

B. B. Rohdendorf. On the Evolution of Flight in Insects 170

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Moscou 1943

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XL, № 4

MATHEMATICS

ON CATEGORIES OF SOME ARC FAMILIES

By L. LUSTERNIK

(Communicated by S. L. Soboleff, Member of the Academy, 31.III.1943)

In his earlier papers (1) the author has dealt with the homological structure of the space $R = R(S_2; a, b)$ of straightened arcs on a surface of type O, having common ends at points a and b. In this paper certain homotopic properties of families of arcs with common ends in S_2, i. e. of sets in R, are to be investigated.

By a deformation in R of a closed family M of arcs from R we shall mean a simultaneous deformation of all arcs of M, such that any two arcs, sufficiently close to each other, will continue as close as we please all along the deformation. By the deformation of a family M of such arcs in R_N we call a deformation such that the arcs of the deformed family have their length never exceeding N throughout the deformation.

A closed family M is said to be of category 1 in R (resp. in R_N), if it can by a deformation in R (resp. in R_N) be reduced to a single arc q in R (resp. in R_N). A family M is said to be of category k in R (resp. in R_N), if it can be broken up into k, but not into $k-1$, closed portions of category 1 in R (resp. in R_N); in this case we shall write

$$\operatorname{cat}_R M = k,$$
$$\operatorname{cat}_{R_N} M = k$$

(this definition can be applied automatically to sets in any space). It may be remarked with regard to sets T_n (2) that with N large enough $\operatorname{cat}_R T_n = \operatorname{cat}_{R_N} T_n$, since any deformation of the polymeridians of T_n in R can with any degree of accuracy be approximated by a deformation in R_N, provided N is sufficiently large.

For closed sets $M \subset R_N$ there takes place the inequality:

$$1 + \operatorname{long}_{R_N} M \leqslant \operatorname{cat}_{R_N} M.$$

This inequality is a generalization of an analogous inequality proved in a paper (3) by S. Froloff and L. Elzholz for the case of compacts R_N. Making use of a theorem established by P. S. Alexandroff (4), concerning the union of a cycle in bicompact spaces, which is a generalization of Pontrjagin's theorem on the removal of the cycle in n-dimensional spaces, it is possible to extend the proof of the inequality discussed in the paper cited to the case of bicompact spaces B.

Namely, let a compact space A in B have the category k.

Then $A = A_1 + A_2 + \ldots + A_k$, $\operatorname{cat}_B A_i = 1$. Assume that $\operatorname{long}_B A \geqslant k$, i. e. A contains a ∇-cycle Z such that there exist $l \geqslant k$ ∇-cycles $Z_1, Z_2, \ldots, Z_l$ such that

$$\varkappa(z, Z_1 \times Z_2 \times \ldots \times Z_l) \neq 0.$$

In virtue of the generalization of Pontrjagin's theorem, Z_1 can be removed from A_1, i. e. there exists a ∇-cycle $\overline{Z}_1$, ∇-homologous to Z_1, located entirely outside A_1. Accordingly, there exists a cycle Z_2, ∇-homologous to Z_2, located entirely outside A_2, and so on, a cycle $\overline{Z}_k$ $(k \leqslant l)$, ∇-homologous to Z_k, located outside A_k (the ∇-cycle is said to be located outside a closed set, if its defining functions are not distinct from zero, unless for a system of arguments taken outside this set). Therefore $\overline{Z}_1 \times \overline{Z}_2 \times \ldots \times \overline{Z}_k$; hence for $l \geqslant k$ $\overline{Z}_1 \times \overline{Z}_2 \ldots \times \overline{Z}_l$ is a forteriori located outside A; on the other hand, the position of z is inside A and, consequently, $0 = \varkappa(z, \overline{Z}_1 \times \overline{Z}_2 \times \ldots \times \overline{Z}_l) = \varkappa(z, Z_1 \times Z_2 \times \ldots \times Z_l)$, which contradicts the assumption.

Since with N large enough $\operatorname{cat}_{R_N} T_n = \operatorname{cat}_R T_n$, we get

$$\operatorname{cat}_R T_n \geqslant \operatorname{long}_{R_N} T_n + 1. \tag{1}$$

In the previous paper [2] it was proved that with N sufficiently large

$$\operatorname{long}_{R_N} T_n = \left(\frac{n+1}{2}\right). \tag{2}$$

So we have

$$(n+1) \geqslant \operatorname{cat}_R T_n \geqslant \frac{n+3}{2}. \tag{3}$$

The right-hand side in (3) follows from (1) and (2).

Now it is to be inferred from elementary considerations that

$$\operatorname{cat}_R T_n \leqslant n+1.$$

No exact value of the category T_n has as yet been obtained.

Thus, we have proved the existence on a sphere of common end arc families of as high a category as we please. For closed curves the existence of high category families is an open question [5].

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MATHEMATICS

ON THE NORMALIZABILITY OF TOPOLOGICAL FIELDS

By I. SHAFAREVICH

(Communicated by I. M. Vinogradow, Member of the Academy, 19.II.1943)

In this paper necessary and sufficient conditions are given in order that a given field be normalizable. These conditions make it possible to reduce some questions of the theory of topological fields to the theory of normalized fields.

In particular, the whole theory of locally bicompact fields can be obtained as a particular case of the theory of normalized fields. Let us say that a topological field is normalizable if we can define a function $\varphi(x)$ of its elements possessing the following properties:

1) $\varphi(x) > 0$, if $x \neq 0$; $\varphi(0) = 0$;
2) $\varphi(x+y) \leqslant \varphi(x) + \varphi(y)$;
3) $\varphi(xy) = \varphi(x)\varphi(y)$;
4) a system of sets $U_\varepsilon\{\varphi(x) < \varepsilon\}$ forms a full system of neighbourhoods of zero.

In order to formulate the conditions of normalizability, the following denotations are necessary. Let R be the set of elements p of the field K such that $p^n \to 0$ for $n \to \infty$, and $\tilde{R}$ be the sets of elements q such that $q^{-1} \notin R$. We shall say that a set W of elements of K is bounded if for any given neighbourhood U of zero there exists a neighbourhood V of zero such that $WV \in U$.

Theorem 1. *In order that a topological field K be normalizable, it is necessary and sufficient that the set R be open and the set $\tilde{R}$ be bounded.*

The necessity of these conditions is obvious. We shall prove their sufficiency by constructing a norm $\varphi(x)$.

Let p be an element of the set R different from zero. Then the sets $p^n R$ $(-\infty < n < +\infty)$ form a full system of neighbourhoods of zero. They increase according as n decreases, and their totality covers K. Let x be any element of the field K different from zero. We denote by $n(x)$ the largest number n, for which we have $x \in p^n R$, and form the set $\alpha_k(x) = \frac{n(x^k)}{k}$. It is easy to prove that

$$\alpha_{k_1}(x) \geqslant \alpha_k(x), \quad \alpha_{k+1}(x) \geqslant \alpha_k(x) - \frac{1}{k+1}.$$

Hence it follows that for $n \to \infty$ $\alpha_k(x)$ tends to a certain limit which will be denoted by $\alpha(x)$.

Now we take an arbitrary number ρ $(0 < \rho < 1)$ and define a function $\psi_\rho(x)$ in the following manner:

$$\psi_\rho(0) = 0,$$
$$\psi_\rho(x) = \rho^{a(x)} \text{ for } x \neq 0.$$

It is easy to see that the function $\psi_\rho(x)$ satisfies to the conditions 1), 3) and 4); however, it is not a norm, since the condition 2) does not hold for any ρ.

A weaker condition

$$2') \quad \psi_\rho(x+y) \leqslant a_\rho \{\psi_\rho(x) + \psi_\rho(y)\}$$

holds instead, a_ρ being fixed. Indeed, if 2') were not fulfilled, the set of the numbers $\frac{\psi_\rho(x+y)}{\psi_\rho(x)+\psi_\rho(y)}$ $(x, y \in K)$ would not be bounded. Then a set z_n would exist such that

$$\frac{\psi_\rho(1+z_n)}{1+\psi_\rho(z_n)} \to \infty \text{ for } n \to \infty.$$

Hence it would follow that $\frac{1}{\psi_\rho(1+z_n)} = \psi_\rho\left(\frac{1}{1+z_n}\right) \to 0$ for $n \to \infty$. But since for $\psi_\rho(x)$ 4) holds, we should have $\frac{1}{1+z_n} \to 0$ for $n \to \infty$. Hence, in virtue of $\frac{\psi_\rho(1+z)}{1+\psi_\rho(z)} = \frac{\psi_\rho(1+z^{-1})}{1+\psi_\rho(z^{-1})}$, it would follow that $\frac{1}{1+z_n^{-1}} \to 0$ for $n \to \infty$, which is impossible in virtue of $\frac{1}{1+z_n} + \frac{1}{1+z_n^{-1}} = 1$.

It remains now to choose such a ρ that $a_\rho \leqslant 1$. This inequality holds if we take $\rho' = \rho^{\frac{1}{a_\rho+1}}$, and thus the proof of the theorem is accomplished.

From Theorem 1 we can obtain some corollaries relating to locally bicompact fields, since, as we can easily prove, the conditions formulated here are satisfied for any field of this kind. This enables us to study the structure of all locally bicompact fields by using the theorems of Ostrowsky (1) on the normalized fields.

In fact, let a locally bicompact field K be connected. In virtue of Theorem 1, it is normalizable; it is easy to see that its norm $\varphi(x)$ is Archimedean.

If K is not connected, then the norm $\varphi(x)$ is non-Archimedean. We even can prove that it is discrete. For, since K is non-Archimedean, the set $R\{\varphi(y) < 1\}$ forms a ring, and the sets $R_\varepsilon\{\varphi(x) < \varepsilon < 1\}$ are its ideals. But since K is locally bicompact, the quotient rings R/R_ε are finite. This contradicts to the hypothesis that between two given numbers ε_1 and ε_2 there exists an infinity of numbers ε_α such that $\varphi(x_\alpha) = \varepsilon_\alpha$ holds for certain elements x_α of K.

Thus, we have proved the following

Theorem 2. *Any locally bicompact field is normalizable. If it is connected, its norm is Archimedean; otherwise it is discrete.*

From Theorem 2 and the theorems by Ostrowsky it follows at once that a connected locally bicompact topological field is a field of real or of complex numbers (2), and that a non-connected locally bicompact topological field is a field of p-adic or of z-adic numbers with a finite coefficient field (3).

As a last application of Theorem 1 we shall consider the relation between two norms $\varphi_1(x)$ and $\varphi_2(x)$ giving the same topology for a field K. If $\varphi(x)$ is the norm constructed with the aid of the process described in the proof of Theorem 1, and if it defines in K the same topology as $\varphi_1(x)$ and $\varphi_2(x)$, we can use an element $p \in R$ and the numbers $\rho_1 < 1$, $\rho_2 < 1$, and obtain

$$\varphi(x) = \varphi_1(x)^{\alpha_1}; \qquad \varphi(x) = \varphi_2(x)^{\alpha_2},$$

where

$$\alpha_1 = \frac{\log \rho_1}{\log \varphi_1(p)}; \qquad \alpha_2 = \frac{\log \rho_2}{\log \varphi_2(p)},$$

whence it follows

$$\varphi_1(x) = \varphi_2(x)^{\beta}, \qquad \beta = \frac{\alpha_2}{\alpha_1}.$$

Thus, we come to

Theorem 3. *If the norms $\varphi_1(x)$ and $\varphi_2(x)$ define the same topology in a field K, they are connected by a relation $\varphi_1(x) = \varphi_2(x)^{\beta}$.*

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ASTRONOMIE

SUR LA POLARISATION DE LA LUMIÈRE RÉFLÉCHIE PAR LA LUNE

Par B. FESSENKOFF, de l'Académie, et O. P. CRAMER

Les observations de Landerer et Barabacheff ont montré que la lumière réfléchie par la Lune est sensiblement polarisée. Cela n'est pas compatible avec les propriétés connues à présent de la surface lunaire qui doit être recouverte d'une couche épaisse de poussière.

Pour vérifier ces résultats nous avons entrepris en été 1941 des observations photographiques de la Lune à l'aide d'une chambre photographique à 70 cm du foyer, munie d'un polaroïde attaché devant l'objectif et susceptible d'être tourné suivant le cercle de position. Le polaroïde a été installé ainsi à 0°, 60°, 120° et à 90°, 150° et 210°. Ainsi on a obtenu un grand nombre d'images photographiques qui peuvent être groupées en deux séries examinées indépendamment.

On peut trouver l'orientation du plan de polarisation par rapport au plan passant par la Lune, le Soleil et la Terre. À l'aide d'une plaque de verre horizontale qui réfléchit la lumière d'une lampe électrique vers l'objectif sous l'angle de la polarisation totale, on trouve d'abord que l'orientation du plan de la polarisation d'un élément quelconque de la surface lunaire est

$$p = P + 150°,$$

P étant la lecture du cercle de position par rapport au zéro arbitraire. Soient maintenant δ_1, δ_2 les déclinaisons de la Lune et du Soleil, l—la distance angulaire entre les deux astres; alors l'angle x du cercle de déclinaison avec l'équateur de l'intensité est donné, comme on peut voir facilement, par la formule

$$\cos x = \frac{\sin \delta_2 - \sin \delta_1 \cos l}{\cos \delta_1 \sin l}.$$

Dans le cas considéré l est voisin à 180°. Par conséquent, l'orientation du plan de polarisation doit dépendre principalement de la direction du rayon-vecteur de l'élément considéré par rapport au centre du disque. En outre, quelles que soient les propriétés réfléchissantes du sol lunaire, le degré de la polarisation au centre du disque au moment de la pleine Lune doit être évidemment zéro. On peut s'attendre à une augmentation continuelle de X vers le bord du disque.

Soient a, b, c les intensités lumineuses d'un certain élément de la surface lunaire, déterminées pour les angles de position 0°, 60°, 120° ou bien 90°, 150°, 210°. Comme j'ai montré ailleurs, le degré de la polarisation X est égal à

$$X = \frac{2\sqrt{a(a-b) + b(b-c) + c(c-a)}}{a+b+c}.$$

L'erreur moyenne ε_X de détermination est

$$\varepsilon_X = \frac{2\sqrt{2(3+\sqrt{3}\,\mathrm{tg}\,2\alpha)}}{1+\sqrt{3}\,\mathrm{tg}\,2\alpha + 2\frac{b}{a}} \cdot \frac{b}{a}\varepsilon,$$

où

$$\frac{b}{a} = \left[\sec 2\alpha - \frac{1}{2} X (1 + \sqrt{3} \operatorname{tg} 2\alpha)\right] (X + \sec 2\alpha)^{-1},$$

ε étant l'erreur moyenne d'une détermination de a, b, c. L'erreur ε dépend de bien des causes. Les principales en sont les inégalités en sensibilité de la couche photographique et les différences de poses. Nous avons préféré par conséquent d'adopter pour a, b, c les brillances relatives aux régions montagneuses de la Lune.

D'autre part, l'angle du plan de polarisation avec la position initiale du polaroïde (lecture du cercle de position étant 0°) est

$$\operatorname{tg} 2\alpha = \sqrt{3} \frac{c - b}{2a - b - c}.$$

Il est à noter que la valeur X est toujours différente de zéro à cause des erreurs inévitables d'observation. En effet, la quantité

$$a(a-b) + b(b-c) + c(c-a)$$

est toujours positive quelles que soient les valeurs a, b, c. Si par example $b > a > c$, on a

$$(a-b)(a-c) + (b-c)^2 > (a-c)^2 > 0.$$

Pourtant les erreurs calculées ε_X sont du même ordre de grandeur que la quantité X. D'autre part, on ne peut constater aucune corrélation entre le degré de la polarisation de la région considérée et sa distance du centre de la Lune, ce que montre bien le tableau suivant:

Distance du centre exprimée en parties du rayon	Degré de polarisation par rapport aux régions montagneuses (valeurs moyennes)		Nombre des observations
	1-er groupe	2-nd groupe	
0.0—0.2	0.056	0.084	1
0.2—0.3	0.034	0.018	2
0.3—0.4	0.042	0.042	5
0.4—0.5	0.034	0.035	6
0.5—0.6	0.034	0.035	7
0.6—0.7	0.040	0.055	7
0.7—0.8	0.032	0.048	7
0.8—0.9	0.019	0.033	3
0.9—1.0	0.043	0.087	5

Il en résulte que la surface lunaire ne montre près du moment de la pleine Lune aucune polarisation sensible.

On peut encore remarquer que la quantité X étant déterminée par une formule de la forme

$$X = \left(\sum a_i^2 - \sum a_{ij}\right)^{\frac{1}{2}} \left(\sum a_i\right)^{-1},$$

il serait plus correct de faire tourner le polaroïde entre les poses consécutives d'un angle suffisamment petit, faisant une partie aliquote de la circonférence. Si la polarisation est nulle, la quantité X, déterminée par la formule généralisée, doit tendre vers zéro quand on combine les valeurs trouvées pour des angles qui font ensemble 60°, 30°, 15° etc.

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HYDRAULICS

FILTRATION FROM BILATERALLY DRAINED CHANNEL OF RECTANGULAR SECTION

By F. B. NELSON-SKORNIAKOV

(Communicated by B. E. Vedeneev, Member of the Academy, 15.IV.1943)

Filtration from a channel of rectangular section is represented schematically in Fig. 1. As the flow is symmetrical with respect to the vertical axis of the channel, it will suffice so study it in the right half of the region Z, i. e. in the region $ABCOED$.

The region of complex velocity $V_x - iV_y = \frac{dW}{dZ}$ for the investigated region of the flow $ABCOED$ is shown in Fig. 2.

By inversion the region $\frac{dW}{dZ}$ is transformed into region $\frac{dZ}{dW}$ represented in Fig. 3. The region $\frac{dZ}{dW}$ is situated on a two-sheet Riemann surface.

The region of complex potential $W = \varphi + i\varphi$ for the flow under investigation is shown in Fig. 4.

Let the region $\frac{dZ}{dW}$ (Fig. 3) be mapped conformably into the region of complex potential $W = \varphi + i\varphi$ (Fig. 4). The function by means of which this transformation is effected, is expressed by the equation

$$\frac{dZ}{dW} = -i\frac{1}{2K} - i\sqrt{\left(\frac{1-m}{2}\right)^2\left[\frac{2+\operatorname{sn}U}{m\operatorname{sn}U-1}\right]^2 - 1} + \\ + \frac{1}{k\pi}\operatorname{ar\,sh}\left[i\frac{2}{(1-m)}\left(\frac{m\operatorname{sn}U-1}{1+\operatorname{sn}U}\right)\right], \qquad (1)$$

where

$$U = \frac{2W}{kH}K - K,$$

where sn is elliptical sine; m, modulus of elliptical functions and integrals; K, full elliptical integral of the first kind; k, coefficient of filtration; H, head, $W = \varphi + i\varphi$, complex potential.

Integration of (1) gives the equation for the characteristic function of the flow:

$$Z = -i\frac{W}{2K} + i\frac{kH}{2K}\int\sqrt{\left(\frac{1-m}{2}\right)^2\left[\frac{1+\operatorname{sn}U}{m\operatorname{sn}U-1}\right]^2 - 1}\cdot dU + \\ + \frac{H}{2\pi K}\int \operatorname{ar\,sn}\left[i\frac{2}{1-m}\left(\frac{m\operatorname{sn}U-1}{1+\operatorname{sn}U}\right)\right]dU + C, \qquad (2)$$

where

$$U = \frac{2W}{kH}K - K. \qquad (3)$$

The total filtration discharge from the channel is given by the equation

$$Q = kH\frac{K'}{K}. \qquad (4)$$

With k and H given it depends solely upon the ratio between the full elliptical integrals of the first kind K and K' for the respective moduli m and $m' = \sqrt{1-m^2}$.

For industrial designs the case when $Q > 4kH$ is of interest. In that case the modulus of elliptical integrals is practically very little different from zero *.

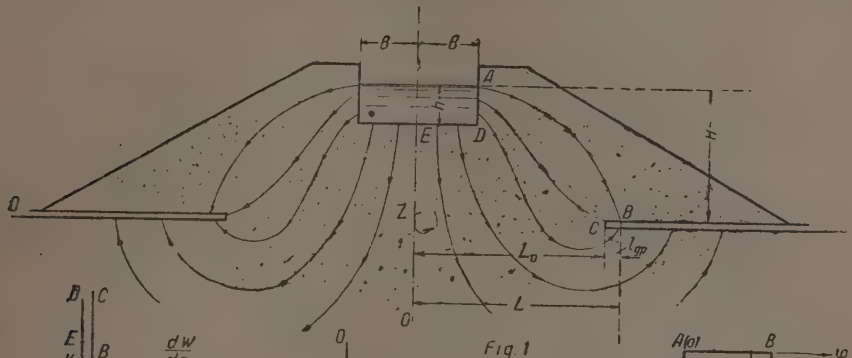


Fig. 1

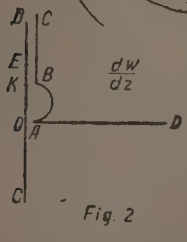


Fig. 2

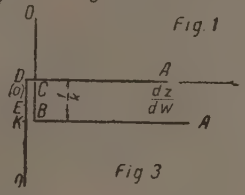
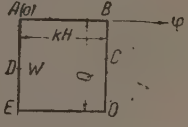


Fig 3



φ' Fig 4

Putting $m \approx 0$, we may write

$$\left.\begin{aligned} &K = \frac{\pi}{2}; \quad K' = \ln \frac{4}{m}; \\ &\left(\frac{1-m}{2}\right)^2 \approx \frac{1}{4}; \quad \operatorname{sn} U = \sin U \end{aligned}\right\} \tag{5}$$

By virtue of (5), equation (2) can be transformed into

$$Z = -\frac{i}{2K} W - i \frac{2kH}{\pi} \int \frac{\sin^4 t - 1}{\sqrt{\sin^4 t - 1}} dt - \\ - i \frac{2H}{\pi} \int \operatorname{arc\,sin} \left(\frac{1}{\sin^2 t}\right) dt + C, \tag{6}$$

where

$$t = \frac{W}{kH} \quad K = \frac{\pi W}{2kH}. \tag{7}$$

Denoting

$$\int \frac{\sin^4 t - 1}{\sqrt{\sin^4 t - 1}} dt = J_1, \tag{8}$$

$$\int \operatorname{arc\,sin} \left(\frac{1}{\sin^2 t}\right) dt = J_2, \tag{9}$$

equation (5) may be rewritten thus:

$$Z = -\frac{i}{2K} W - i \frac{2kH}{\pi} J_1 - i \frac{2H}{\pi^2} J_2. \tag{10}$$

* The normal cross-section of the channel is characterized by the parameters: width of water level $2B > 4H$; depth of water in the channel $h > \frac{H}{2}$. Under these conditions $Q/K > 4H$. Thus, for $m = 0.01$ we find from the tables of elliptical integrals that $\frac{K'}{K} = 3.814$ and $\frac{Q}{K} = 3.814\ H$.

The integral J_1 by means of the substitution $\sin t = x$ is reduced to an elementary integral, and the integral J_2 is computed by approximate methods.

Writing the equation (9) for the points B, D and E, we obtain a system of equations, by means of which the parameters in the region of flow Z are defined. In all, there are four parameters in the region Z: h, H, B and L. Three of them being given, one may determine the fourth.

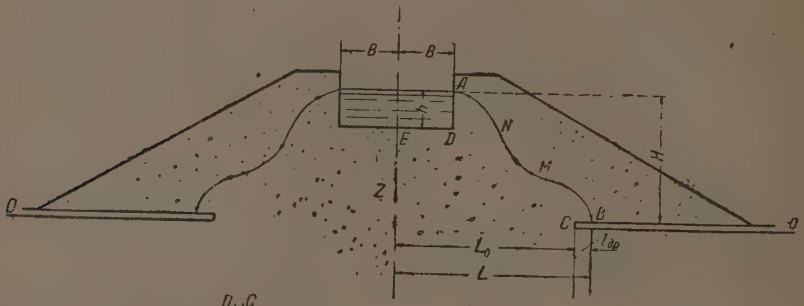


Fig 5.

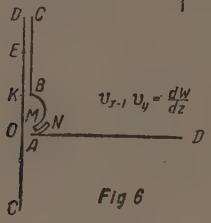


Fig 6

The equation for the free surface is obtained from (10) by putting $W = ky$ and $Z = x + iy$. The parameter l_{qp} is determined from the condition that at point C $\left|\frac{dZ}{dW}\right|_C = 0$.

Generally speaking, the hydromechanic solution obtained is not unique. Indeed, we have taken the region $\frac{dZ}{dW}$ in accordance with Fig. 2. But other outlines of that region are also possible physically, so that the free surface of the ground flow AB may have one or several points of inflexion (Fig. 5).

With a certain combination of parameters h, L, B, and H in the region of flow Z, the region $\frac{dZ}{dW}$ will assume the shape represented in Fig. 6. This shape of the region $\frac{dZ}{dW}$ differs from that of Fig. 2 in including ANM in addition. If, therefore, we supposed the region $\frac{dZ}{dW}$ to accord with Fig. 6, we would obtain a different result.

In particular, the region $\frac{dZ}{dW}$ may be shaped as in Fig. 6 when the width of the channel B is small in comparison with the head H and the parameter L. Such channels, however, are exceptions in practice. In literature no indications as yet have been made to the effect that one or several points of inflexion of the free surface of a ground flow are possible in the case of filtration from channels. It would seem concerning the filtration from channels that of all of the hydromechanic solutions obtained by means of Kirchhoff's complex, without allowing for the possible existence of inflexion points of the free surface, none can claim to be the only possible solution physically. Later this problem will be treated in detail.

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MATHEMATICAL PHYSICS

RELATION BETWEEN OPERATIONAL EXPRESSIONS OF TWO ARBITRARY FUNCTIONS AND THE OPERATIONAL REPRESENTATION OF THEIR PRODUCT*

By G. A. GRÜNBERG

(Communicated by A. F. Joffe, Member of the Academy, 5.IV.1943)

In a discussion on non-linear problems that recently took place, A. A. Kharkevich expressed the idea that new ways for the application of the operational calculus to non-linear problems and to problems reducible to linear equations with variable coefficients might possibly be opened up by establishing a relation between operational expressions of two arbitrary functions and the operational representation of their product.

In this work it is shown that such a relation is given by the following general theorem:

If

$$\left.\begin{aligned} \overline{f} \equiv f(p) &= \int_0^\infty e^{-pt} f(t)\,dt, \\ \overline{\varphi} \equiv \overline{\varphi}(p) &= \int_0^\infty e^{-pt} \varphi(t)\,dt \end{aligned}\right\} \quad (p \text{ is a complex number}) \qquad (1)$$

are the operational expressions («images») of two functions $f(t)$ and $\varphi(t)$, these functions satisfying certain general conditions, there takes place the relation

$$\overline{(f\varphi)} = \int_0^\infty e^{-pt} f(t)\,\varphi(t)\,dt = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} \overline{f}(w)\,\overline{\varphi}(p-w)\,dw, \qquad (2)$$

where a is a positive number, suitably chosen, and the real part of p should satisfy certain conditions which are made plain by the following proof of the theorem.

Proof. Let $F(t)$ and $\Phi(t)$ satisfy the conditions of Dirichlet and an additional condition that $\int_{-\infty}^{+\infty} |F(t)|\,dt$ and $\int_{-\infty}^{+\infty} |\Phi(t)\,dt$ are finite. Then, according to a well-known theorem of Rayleigh (1), there takes place the relation

$$\int_{-\infty}^{+\infty} F(t)\,\Phi(t)\,dt = \pi \int_0^{+\infty} [MP+NQ]\,d\nu \qquad (3)$$

* The operational representation is to be understood in the sense of Heaveside, see formula (1). The notation adopted in this paper for the «images» of functions differs from that usually given in textbooks on operational calculus, i. e. from $\varphi^* = p\int_0^\infty e^{-pt}\varphi(t)\,dt$, in that the fac-

where

$$\left.\begin{aligned} M+iN &= \frac{1}{\pi}\int_{-\infty}^{+\infty} e^{\pm i\nu\tau} F(\tau)\,d\tau, \\ P+iQ &= \frac{1}{\pi}\int_{-\infty}^{+\infty} e^{\pm i\nu\tau} \Phi(\tau)\,d\tau. \end{aligned}\right\} \tag{4}$$

Suppose that for $t<0$ $F(t)=0$ and $\Phi(t)=0$. Equations (3) and (4) give

$$\int_0^\infty F(t)\,\Phi(t)\,dt = \pi\int_0^\infty [MP+NQ]\,d\nu. \tag{5}$$

Here

$$\pi(M \pm iN) = \int_0^\infty e^{\pm i\nu\tau} F(\tau)\,d\tau = F(\pm i\nu) \tag{6}$$

and

$$\pi(P \mp iQ) = \int_0^\infty e^{\mp i\nu\tau} \Phi(\tau)\,d\tau = \overline{\Phi}(\pm i\nu). \tag{7}$$

From this we find that

$$\pi^2(MP+NQ) = \frac{1}{2}\left\{\overline{F}(i\nu)\,\Phi(-i\nu) + \overline{F}(-i\nu)\,\overline{\Phi}(i\nu)\right\}. \tag{8}$$

Substitution in (5) gives

$$\int_0^\infty F(t)\,\Phi(t)\,dt = \frac{1}{2\pi}\left\{\int_0^\infty \overline{F}(i\nu)\,\overline{\Phi}(-i\nu)\,d\nu + \int_0^\infty \overline{F}(-i\nu)\,\overline{\Phi}(i\nu)\,d\nu\right\}. \tag{9}$$

On the right-hand side we put $i\nu=z$ in the first integral, and $-i\nu=z$ in the second. Thus, we obtain:

$$\int_0^\infty F(t)\,\Phi(t)\,dt = \frac{1}{2\pi i}\left\{\int_0^{\infty i} \overline{F}(z)\,\overline{\Phi}(-z)\,dz - \int_0^{-i\infty} \overline{F}(z)\,\overline{\Phi}(-z)\,dz\right\} =$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} \overline{F}(z)\,\overline{\Phi}(-z)\,dz. \tag{10}$$

This form of Rayleigh's theorem has been given by Rimsky-Korsakov (2)*.

Suppose now that $f(t)$ and $\varphi(t)$ are two functions such that the products $e^{-at}f(t)$ and $e^{-bt}\varphi(t)$ satisfy the conditions imposed upon $F(t)$ and $\Phi(t)$, a, b being positive constants. Putting then $F(t)=e^{-at}f(t)$, $\Phi(t)=e^{-(p-a)t}\varphi(t)$, where p is a complex number such that $\mathrm{Re}(p-a)\geqslant b$, i. e. $\mathrm{Re}(p)\geqslant a+b$, and noticing that

$$\overline{F}(z)=\overline{f}(z+a), \qquad \overline{\Phi}(-z)=\overline{\varphi}[p-(z+a)],$$

---

tor p is absent. This difference is to be taken into account on comparing our operational formulae with the usual formulae. The customary notation mainly owes its use to tradition and is no more convenient than ours, except in the case when those solutions of the differential equation are to be found, which correspond to zero initial conditions and constant external forces. On the other hand, our notation has the advantage of a perfect symmetry between the direct and inverse formulae, cf. formula (2) with a well-known theorem of Borel, in which the function, whose «image» is the product of the «images» of two other functions, is expressed through the latter.

* As Rimsky-Korsakov takes usual denotations for the «images», his formula by its aspect is somewhat different from our formula (10).

we obtain from (10)

$$\int_0^\infty e^{-pt} f(t)\,\varphi(t)\,dt = \overline{(f\varphi)} = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} \bar{f}(z+a)\,\bar{\varphi}[p-(z+a)]\,dz =$$

$$= \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \bar{f}(w)\,\bar{\varphi}(p-w)\,dw. \qquad (11)$$

This is the formula required.

Example. Let $f(t)=e^{-\alpha t}$, $\varphi(t)=e^{-\beta t}$, $\alpha>0$, $\beta>0$; then

$$f(w)=\frac{1}{\alpha+w}, \quad \bar{\varphi}(p-w)=\frac{1}{p+\beta-w}$$

and, as in that case we may put $a=b=0$, $\mathrm{Re}(p)>0$, we shall obtain from (11)

$$J=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} \bar{f}(w)\,\bar{\varphi}(p-w)\,dw=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{dw}{(\alpha+w)(p+\beta-w)}.$$

If the path of integration is supplemented in the left half-plane by a circular arc of infinite radius (the integral over this arc is zero), J is reduced to the residue at point $w=-\alpha$*. The value of the integral is therefore

$$J=\frac{1}{\alpha+\beta+p}$$

and is equal in fact to the quantity

$$\overline{(f\varphi)}=\int_0^\infty e^{-pt}e^{-\alpha t}e^{-\beta t}dt=\int_0^\infty e^{-(\alpha+\beta+p)t}\,dt.$$

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* The point $w=p+\beta$ because of $\mathrm{Re}(w)>0$ lies outside the contour.

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GEOPHYSICS

ON THE PROBLEM OF UNSTATIONARY DISTRIBUTION OF AIR TEMPERATURE NEAR THE UNDERLYING SURFACE

By M. SHVEZ

(Communicated by N. E. Kochin, Member of the Academy, 3.III.1943)

Let the influx of radiant energy to unit earth surface be known as a function of time. This influx consists of long-wave radiation a directed towards the underlying surface, and of short-wave radiation s.

The air temperature in the boundary layer is determined from the equation of heat influx:

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial z}k\frac{\partial T}{\partial z}+f(z,t), \qquad (1)$$

where T is temperature; t, time; z, vertical coordinate; k, coefficient of turbulent temperature conductivity; f, influx of heat (per unit time to unit volume) from thermic radiation.

Let T_1 be the temperature of the underlying medium at whose surface absorption and radiation of heat take place. Along with equation (1) we must write down the equation of heat conductivity for the underlying medium *

$$\frac{\partial T_1}{\partial t}=\frac{\partial}{\partial z}k_1\frac{\partial T}{\partial z}. \qquad (2)$$

As boundary conditions will serve:

1) The equation of heat balance at the underlying surface ($z=0$):

$$-\lambda\frac{\partial T}{\partial z}+\lambda_1\frac{\partial T_1}{\partial z}+\sigma T^4=a+s+rk\frac{\partial q}{\partial z}, \qquad (3)$$

where $\lambda=c_p\rho k$ is the coefficient of the turbulent heat conductivity of the air; $\lambda_1=c_1\rho_1 k_1$ is the coefficient of heat conductivity of the underlying medium; ρ and ρ_1 are air density and density of the underlying medium, respectively; σ is the constant of Stephan-Boltzman; r, latent heat of evaporation; q, specific humidity; $-rk\frac{\partial q}{\partial z}$, flux of heat lost on evaporation.

2) Temperature equality with $z=0$:

$$T(0,t)=T_1(0,t). \qquad (4)$$

Temperature variations being usually small as compared with the mean absolute temperature of the underlying surface ϑ **, we may assume, in order to make the problem linear, that

$$T=\vartheta+\tau; \qquad T_1=\vartheta+\tau_1.$$

* If the underlying medium is a sea or a lake, k_1 will be the coefficient of turbulent temperature conductivity.

** ϑ may also be taken as the temperature of the underlying surface at the initial moment of time.

In the first approximation the influx of heat from radiation can be neglected; the problem will then be reduced to the solution of the equations:

$$\frac{\partial \tau}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial \tau}{\partial z}, \tag{1'}$$

$$\frac{\partial \tau_1}{\partial t} = \frac{\partial}{\partial z} k_1 \frac{\partial \tau_1}{\partial z}, \tag{2'}$$

the boundary conditions being:

$$\lambda \frac{\partial \tau}{\partial z} - \lambda_1 \frac{\partial \tau_1}{\partial z} - 4\sigma\vartheta^3\tau = -rk\frac{\partial q}{\partial z} - a - s + \sigma\vartheta^4 = \Psi(t) \tag{3'}$$

$$\tau(0, t) = \tau_1(0, t), \tag{4'}$$

The coefficient of turbulent heat conductivity $\lambda = c_p \rho k$ is characterized by that it approaches zero near the underlying surface, after which up to a certain point it shows an approximately linear increase. Let us assume that $k = cz^n (n < 1)$. Coefficient k_1 may with a sufficient degree of accuracy be considered constant.

Let $-\lambda_1 \frac{\partial \tau_1}{\partial z}\Big|_{z=0} = \varphi(t)$; the solution of equation (1') will then assume the shape:

$$\tau_1(z, t) = A \int_0^t \frac{\varphi(\alpha)}{\sqrt{t-\alpha}} \exp\left[-\frac{z^2}{4k_1(t-\alpha)}\right] d\alpha, \tag{5}$$

where $A = \frac{2}{c_1\rho_1\sqrt{\pi k_1}}$. It is obvious that

$$\tau_1(0, t) = A \int_0^t \frac{\varphi(\alpha)}{\sqrt{t-\alpha}} d\alpha. \tag{5'}$$

Let

$$\lim_{z\to 0} \lambda \frac{\partial \tau}{\partial z} = F(t). \tag{6}$$

The solution of equation (1) will be sought for in the form:

$$\tau = \frac{1}{t^p} f\left(\frac{z^{\frac{1}{p}}}{ct}\right) = \frac{1}{t^p} f(\eta).$$

Substituting this expression into (1'), we obtain for the determination of $f(\eta)$ the usual differential equation:

$$\frac{d}{d\eta}\eta^p \frac{df}{d\eta} + p^2\eta^p \frac{df}{d\eta} + p^3\eta^{(n-1)p} f = 0.$$

As a particular solution of this equation serves $f = \exp\left[-\frac{p^2 z^{\frac{1}{p}}}{ct}\right]$, $\left(p = \frac{1}{2-n}\right)$. Consequently,

$$\tau = \frac{1}{t^p} \exp\left(-\frac{p^2 z^{\frac{1}{p}}}{ct}\right). \tag{7}$$

From (7) the following general solution («integral from sources») of equation (1) is obtainable, satisfying to the boundary condition (6):

$$\tau(z, t) = B \int_0^t \frac{F(\alpha)}{(t-\alpha)^p} \exp\left[-\frac{p^2 z^{\frac{1}{p}}}{c(t-\alpha)}\right] d\alpha, \tag{8}$$

where $B = \frac{p^{2p-1}}{c^p \rho c_p} \frac{1}{\Gamma(p-1)}$ (Γ is gamma function). It is apparent that

$$\tau(0, t) = B \int_0^t \frac{F(\alpha)}{(t-\alpha)^p} d\alpha.$$

The boundary condition (4′) yields:

$$A\int_0^t \frac{\varphi(\alpha)}{\sqrt{t-\alpha}}\,d\alpha = B\int_0^t \frac{F(\alpha)}{(t-\alpha)^p}\,d\alpha. \qquad (4'')$$

Denoting $4c\vartheta^3=\mu$, we get instead of the equation of heat balance

$$F(t)+\varphi(t)-\mu B\int_0^t \frac{F(\beta)}{(t-\beta)^p}\,d\beta = \Psi(t). \qquad (9)$$

Let us multiply equation (9) by $\frac{d\alpha}{\sqrt{t-\alpha}}$, after having substituted t by α; integrating hereupon the result obtained along α within the interval from 0 to t, we have:

$$\int_0^t \frac{F(\alpha)}{\sqrt{t-\alpha}}\,d\alpha+\int_0^t \frac{\varphi(\alpha)}{\sqrt{t-\alpha}}\,d\alpha-\mu\beta\int_0^t \frac{d\alpha}{\sqrt{t-\alpha}}\int_0^\alpha \frac{F(\beta)}{(\alpha-\beta)^p}\,d\beta=\int_0^t \frac{\Psi(\alpha)}{\sqrt{t-\alpha}}\,d\alpha. \qquad (10)$$

According to the well-known formula of Dirichlet:

$$\int_0^t \frac{d\alpha}{\sqrt{t-\alpha}}\int_0^\alpha \frac{F(\beta)}{(t-\beta)^p}\,d\beta=\int_0^t F(\beta)\,d\beta\int_\beta^t \frac{d\alpha}{(\alpha-\beta)^p\sqrt{t-\alpha}}=\int_0^t F(\alpha)\,K(\alpha-t)\,d\alpha.$$

It is rather easy to find out that $K(t-\alpha)=(t-\alpha)^{1/2-p}\,D(1-p,\ {}^1/_2)$, where

$$D(1-p,\ {}^1/_2)=\frac{\Gamma(1-p)\,\Gamma({}^1/_2)}{\Gamma({}^3/_2-p)}.$$

Considering (4″), expression (10) can be represented as follows:

$$\int_0^t \frac{F(\alpha)}{\sqrt{t-\alpha}}\,d\alpha+\frac{B}{A}\int_0^t \frac{F(\alpha)}{(t-\alpha)^p}\,d\alpha-\mu B\int_0^t F(\alpha)\,K(\alpha-t)\,d\alpha=\int_0^t \frac{\Psi(\alpha)}{\sqrt{t-\alpha}}\,d\alpha.$$

It is clear that $A\int_0^t \frac{\Psi(\alpha)}{\sqrt{t-\alpha}}\,d\alpha$ is the temperature of the underlying surface for the case when there is no heat conductivity into the air. Let us denote this temperature by $\vartheta_1(t)$. Now for determining $F(\alpha)$ we have the integral equation

$$\int_0^t \left[A(t-\alpha)^{-\frac{1}{2}}+B(t-\alpha)^{-p}-BA\mu D(t-\alpha)^{-p+\frac{1}{2}}\right]F(\alpha)\,d\alpha=\vartheta_1(t).$$

Let us denote $t-\alpha=x$; the kernel of this equation $R(x)$ is transformed into infinity with $x=0$. Whatever the accuracy required, we can express $R(x)$ in the shape:

$$R(x)=x^{-\nu}(a_0+a_1x+\ldots+a_nx^n).$$

The solution of the integral equation (1) is then obtained from:

$$F(t)=\frac{\sin\nu\pi}{\pi}\int_0^t \vartheta_1'(\alpha)\,L_1(t-\alpha)\,d\alpha,$$

in which

$$L_1(x)=\frac{x^{\nu-1}}{a_0}+\frac{\alpha^{n-\nu}}{Q'(\alpha)}\gamma_\nu(\alpha,x)+\frac{\beta^{n-\nu}}{Q'(\beta)}\gamma_\nu(\beta,x)+\ldots+\frac{\lambda^{n-\nu}}{Q'(\lambda)}\gamma_\nu(\lambda,x),$$

where $\alpha,\beta,\ldots,\lambda$ are the roots of the algebraic equation

$$Q(x)=a_0x^n+(1-\nu)\,a_1x^{n-1}+(1-\nu)(2-\nu)\,a_2x^{n-2}+\ldots+(1-\nu)(2-\nu)\ldots(n-\nu)\,a_n=0;$$

$\gamma_\nu(x)$ denotes the incomplete gamma function:

$$\gamma_\nu(x) = e^x \int_0^x y^{\nu-1} e^{-y} dy,$$

the values of which have been tabulated [2]. Instead of (10) the equation of heat balance can also be written down in the following way:

$$F(t) + \varphi(t) - \mu A \int_0^t \frac{\varphi(\alpha)}{\sqrt{t-\alpha}} d\alpha = \Psi(t).$$

Let us multiply both parts of the correlation by $\frac{d\alpha}{(t-\alpha)^p}$, having previously replaced t by the letter α; let us next integrate the result of the multiplication along α from 0 to t; we have then

$$\int_0^t \frac{F(\alpha)}{(t-\alpha)^p} d\alpha + \int_0^t \frac{\varphi(\alpha)}{(t-\alpha)^p} d\alpha + A\mu \int_0^t \frac{d\alpha}{(t-\alpha)^p} \int_0^\alpha \frac{\varphi(\beta)}{\sqrt{t-\beta}} d\beta = \int_0^t \frac{\Psi(\alpha)}{(t-\alpha)^p} d\alpha. \quad (11)$$

According to Dirichlet's formula:

$$\int_0^t \frac{d\alpha}{(t-\alpha)^p} \int_0^\alpha \frac{\varphi(\beta)}{\sqrt{t-\beta}} d\beta = \int_0^t \varphi(\beta) d\beta \int_\beta^t \frac{d\alpha}{(\alpha-\beta)^p \sqrt{t-\alpha}} = \int_0^t \varphi(\alpha) K(\alpha - t) d\alpha.$$

Taking into account (4″), we obtain for the determination of $\varphi(t)$ Volterra's integral equation of the 1st kind:

$$\int_0^t [B(t-\alpha)^{-\frac{1}{2}} + A(t-\alpha)^{-p} - \mu ABD(t-\alpha)^{-p+\frac{1}{2}}] \varphi(\alpha) d\alpha = \Theta(t),$$

where

$$\Theta(t) = B \int_0^t \frac{\Psi(\alpha) d\alpha}{(t-\alpha)^p}.$$

It is evident that

$$\varphi(t) = \frac{\sin \nu\pi}{\pi} \int_0^t \Theta'(\alpha) L(t-\alpha) d\alpha.$$

The calculations involved in the solution of the problem we have set before ourselves become much simpler in case radiation balance is known. The flux of heat consumed on moisture evaporation is supposed here to be known, as well as a. The way to determine this influx of heat will form the subject of our next communication. We shall also introduce a correction into the air temperature at the expense of radiant heat exchange, in other words, give a solution of the unsimplified equation (1).

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CHEMISTRY

ABSORPTION SPECTRA OF COMPLEX COMPOUNDS

CIS-TRANS-ISOMERISM OF COMPLEX PLATINUM COMPOUNDS OF THE DIAMINE SERIES AND LIGHT ABSORPTION

By A. V. BABAEVA

(Communicated by V. G. Chlopin, Member of the Academy, 16.II. 1943)

The chemistry of complex compounds is very rich in isomeric forms. At present in order to prove the presence of this or that isomer, both chemical and physical methods are used. Among the latter the measurement of dipole moments, X-ray photographs of crystals and spectra of combination scattering are used very successfully. It should be expected that cis-trans-isomerism should also be reflected in the visible and in the ultra-violet spectroscopy. The difference in the colour of the cis-trans-isomers testifies to the lawfullness of such a conjecture.

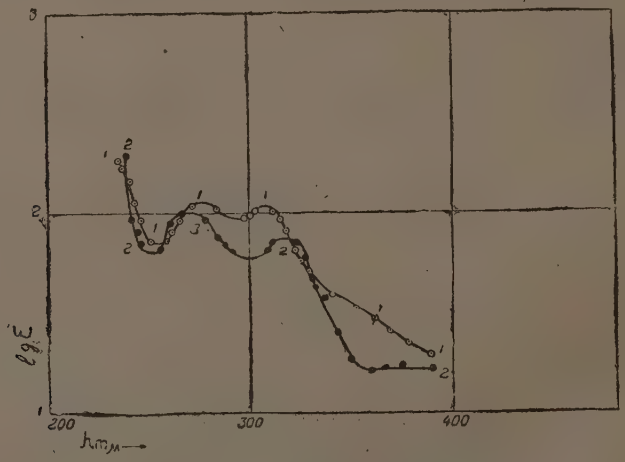

Fig. 1. *1.* $[(NH_3)_2 Pt + Cl_2]$ - cis. *2.* $[(NH_3)_2 PtCl_2]$ - trans

The curves of the absorption of light by complex isomeric compounds of cobalt and chromium have been investigated by many authors. The results of these investigations are contradictory. Thus, Ley and Winkler (1) studied the absorption of light by isomeric cobalt ethylenediamine dichlorides [Co en_2 Cl_2] X and affirm that steric isomerism influences but little the absorption spectrum of these compounds. Schibata (2) had investigated the light absorption curves of the isomeric [Cr en_2 NH_3 Br] Br_2 and [Co en_2 NH_3 Br] Br_2 and found the character of the spectrum of the cis- and trans-form to be the same, the lg ε of the minimum of the absorption bands of the trans-compounds having, however, a lower value, compared to that of cis-compounds. The logarithms of the molecular coefficients of absorption

for the maxima of the two isomeric forms were found to be the same. Uspenski and his collaborators (3), on the basis of the investigation of absorption spectra of a number of isomeric complex compounds of cobalt, came to a different conclusion, viz. that the absorption of light in the visible area depends upon the spectral configuration of the complex. The authors distinguish two types of isomeric compounds. The first type is characterized by the cis-isomer having a maximum of absorption at those wave-lengths at which for the trans-forms there is a minimum. The second type gives cis-isomers that are optically very similar to trans-isomers and show but little difference in the magnitude of the absorption coefficients.

Luther and Nikopolos (4) have investigated the absorption spectra of the isomeric $[Co(NH_3)_4Cl_2]X$ and $[Co(NH_3)_4(NO_2)_2]X$ and affirm that the maxima of the

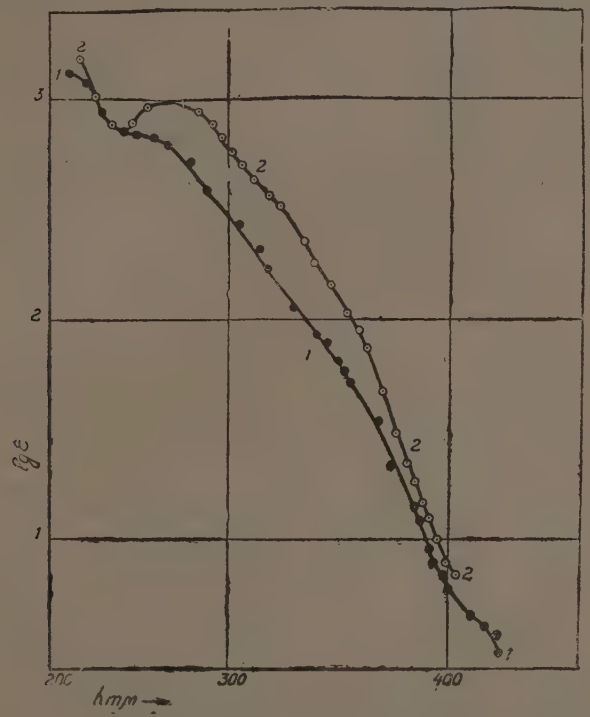

Fig. 2. *1.* $[(NH_3)_2NO_2ClPt]$ - cis. *2.* $[(NH_3)_2NO_2ClPt]$ - trans

absorption bands of the trans-forms are somewhat shifted in the direction of the long-wave end of the spectrum compared to the maxima of the cis-forms. This shift is the more considerable, the stronger the lability of the groups participating in the isomerism. The same effect has been observed by Czegledy on the absorption curves of the isomeric amino-aquochlorides of cobalt.

Schibata (5) supposes that the absorption spectra of the trans-isomers of complex compounds are characterized by the presence of a third absorption band which is to be found in the short-wave part of the ultra-violet*. He thinks that the third band in the case of cobalt nitroamines is due to the trans-configuration of nitro-radicals. Tsuchida (6) repeated and supplemented the experiments of Schibata and widened the explanation of the third band given by that author. The third band;

* Concerning the number of bands in the absorption spectra of complex compounds see one of the former papers by the author, C. R. Acad. Sci. URSS, XL, No. 2 (1943).

according to him, is due to a couple or to couples of negative substituents of coordinated trans-positions.

In our first preliminary communication dealing with the investigation of the absorption of light by complex compounds of platinum (7) it was pointed out that in the absorption spectra of the isomers of the diamine series of bivalent platinum the author was unable to discover effects giving the possibility of distinguishing isomeric compounds. However, it was thought probable that cis- and trans-isomers that were different chemically could not but be different optically either. Therefore the experiments were repeated many times, the method of investigation was considerably modified by using the sector method of Scheibe.

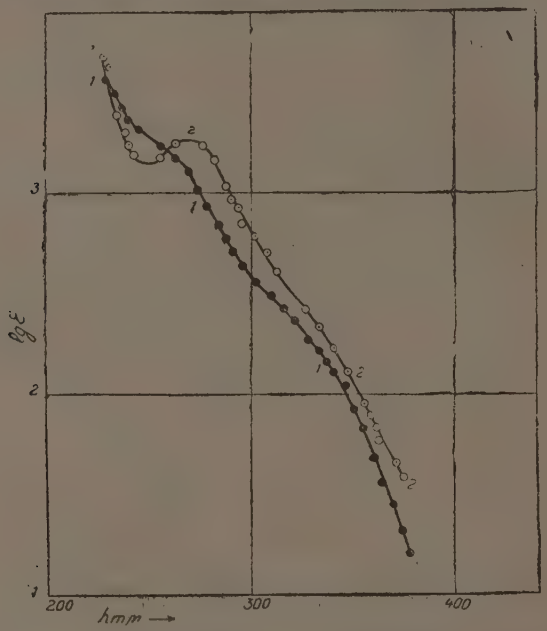

Fig. 3. *1.* $[(NH_3)_2(NO_2)_2Pt]$ - cis. *2.* $[(NH_3)_2(NO_2)_2Pt]$ - trans

Solutions of Peyronet salt $[(NH_3)_2Cl_2Pt]$ and of the isomeric salt of the II Reiset, cis- and trans-diamino-nitrochlorides $[(NH_3)_2NO_2ClPt]$ and of diamino-dinitrates $[(NH_3)_2(NO_2)_2Pt]$ were investigated. They ranged within a concentration from 0.01 to 0.0001 mol. and were photographed every 30—40 min. after their preparation. The interval investigated ranged from 600 to 220 mμ.

	λ_1 mμ	lg ε_1	λ_2 mμ	lg ε_2	λ_3 mμ	lg ε_3
1. $[(NH_3)_2Cl_2Pt]$ -cis	366	1.70	308	2.06	273	2.09
2. $[(NH_3)_2Cl_2Pt]$ -trans	376	1.24	312	1.87	273	1.89
3. $[(HN_3)_2NO_2ClPt]$ -cis			(252)	(2.84)		
4. $[(NH_3)_2NO_2ClPt]$ -trans			274	2.98		
5. $[(NH_3)_2(NO_2)_2Pt]$ -cis			(246)	(3.32)		
6. $[(NH_3)_2(NO_2)_2Pt]$ -trans			270	3.26		

As may be seen both from the table and from the curves (Figs. 1, 2, 3), the first two compounds give in the investigated range of wave-length three absorption

bands. The one corresponding to the longest wave, the first absorption band, is rather diffuse in the case of both isomers. The second and third bands are more or less clear-cut. In the case of trans-diamino-dichloride the maxima of the first and the second band are shifted by an almost equal number of wave-lengths (9—10 mμ) towards the long-wave end of the spectrum as compared to the maximum of the cis-isomer. The position of the maximum of the third band is the same for both isomers.

In the absorption spectrum of the isomeric $[(NH_3)_2\ NO_2\ Cl\ Pt]$ and $[(NH_3)_2(NO_2)_2\ Pt]$ there is only one broad band. Probably, this band is the second absorption band. Its maximum for trans-compounds compared with that of the cis-isomer is shifted 22—24 mμ towards the long-wave side and expressed more clearly than in the case of the cis-form. Such a position of the maxima is evidence that the cis-compounds are more stable photochemically than the trans-isomers.

The comparison of the position of the maximum of the second band for different compounds of analogous configuration shows that actually with a decrease of the substituents' lability the maximum is shifted to the region of short wave-lengths. Thus, chlorine is more labile than NO_2, and therefore the corresponding bands for the amino-nitrochlorides are at shorter wave-lengths compared to those for diamino-dichlorides, and the presence of two nitrogroups makes the complex so much more stable that this second band is found at the lowest frequencies.

The results of our investigation lead us to the conclusion that if the absorption curve is no sufficient ground in itself for decisions as to which isomeric form is present, nevertheless the question can be solved by comparing the absorption curves for the two isomers.

Conclusions

1. The absorption of light in the region of 600 to 220 mμ for three pairs of isomers $[(NH_3)_2Cl_2Pt]$, $[(NH_3)_2NO_2ClPt]$, $[(NH_3)_2(NO_2)_2Pt]$ has been investigated.
2. It has been demonstrated that the cis-trans-isomery of platodiamines is reflected in the position of the absorption bands. The second band in the absorption spectrum of the trans-compounds of this series is shifted towards the long-wave end of the spectrum as compared to the same band of cis-isomers.

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CHEMISTRY

FORMATION OF COAL DENDRITES BY CATALYTIC DECOMPOSITION OF ALCOHOLS ON METALS

By A. A. BALANDIN and V. V. PATRIKEEV

(Communicated by V. G. Chlopin, Member of the Academy, 20. III. 1943)

According to a viewpoint put forward recently, the decomposition of certain carbon compounds, while in contact with metal surfaces, gives rise to a formation of coal in the form of dendrites (1). This hypothesis has supplied a satisfactory explanation of some facts of kinetic nature observed during catalytic decomposition of cyclohexane on nichrome, platinum, iron (1) and of ethylbenzene on iron (2).

Fig. 1

Fig. 2

At present decisive evidence has been obtained by us that coal resulting from catalytic decomposition of alcohols on metals is deposited in the form of dendrites. This is seen from the following example.

A constantan wire (60% of copper and 40% of nickel), 0.35 mm in diameter, was stretched along the axis of a pyrex glass tube 20 mm in diameter. The air was forced out of the tube by carbon dioxide, and isopropyl alcohol vapour was made to pass through the tube at 450° C at a rate of 0.23 ml of liquid alcohol per minute during an interval of half an hour. At the end of the run and after the cooling of the electric furnace in which the heating was effected, the tube was taken out. On the wire soot was found to have been deposited in the shape of dendrites. These dendrites are distinctly visible in a magnified photograph (Fig. 1). Some of the dendrites were more than 1 mm long. This may easily be seen by comparing their length with the diameter of the wire where the latter is free from soot (the right-hand side of Fig. 1). The layer of soot came off as a result of shaking, which shows that it does not adhere firmly to the wire.

In alcohol experiments a phenomenon was observed differing from those with cyclohexane. On the photograph (Fig. 2) the soot is seen to come off at some places, leaving the surface of the metal bare. The separating portion has the form of a brush, the base from which the dendrites are growing presenting a very thin plate, sometimes reaching 20 mm in length, and having a metallic brown lustre. Upon treatment with nitric acid and with excessive ammonia the plates were found to contain copper, while no copper was found in the dendrites.

A possible way to explain the phenomenon is that a carbide film is at first formed on the metal, and is then decomposed with the formation of the metal in a more loose form than the surface of the wire. From here the dendrites start growing. The crystals of the deposited metal being smaller in size and taking up a larger total volume than those of the wire metal, this gives rise to exfoliation. Hence it follows that a similar process of dispersion of iron in coal formed during the decomposition of gasoline, observed by Hofmann([3]), actually falls into two distinct processes.

The dendrites do not consist of pure carbon, but contain also hydrogen and oxygen. The elementary composition of the soot obtained in our runs corresponds roughly to the empirical formula $(C_{23}H_4O)_x$.

5.62 mg of the substance: 19.23 mg CO_2; 0.72 mg H_2O
5.23 mg of the substance: 17.96 mg CO_2; 0.60 mg H_2O.
No ash. Found %: C—93.32; 93.63; H—1.43; 1.28.
$(C_{23}H_4O)_x$. Calculated %: C—93.25; H—1.36.

The results obtained may be interpreted as follows. The soot has the lattice of graphite, and the axis of a dendrite should run parallel to the *c*-axis, as it is the unique direction not repeated by virtue of symmetry. We consider the layers perpendicular to the *c*-axis as having the structure of a condensed aromatic hydrocarbon with 6-member cycles, the hydrogen atoms in the CH groups being located only on the periphery [compare the view expressed by Lowry ([4])]. Their presence may explain the inhibition of the growth of dendrites in this plane and their growth along the *c*-axis.

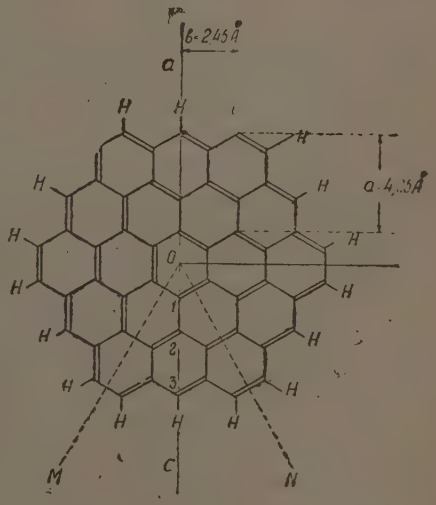

Fig. 3

In Fig. 3 a crystalline layer in the form of a regular hexahedron is seen to be filled up by elementary hexahedra with sides of 1.41 Å as in graphite ([5]). As a matter of fact, the sides are somewhat smaller, though larger than in benzene, 1.39 Å. The benzene nucleus is located in the centre. The distribution of double bonds in separate parts corresponds to one of the resonance forms of graphite according to Pauling ([5]), and in the other resonance forms the number of H atoms suffers no change.

Let us evaluate the number z of cycles corresponding to the diameter of Fig. 3, and the length of the diameter in the direction of the a-axis. The ratio of the number of carbon atoms to the number of hydrogen atoms is on account of symmetry the same in the sector *MON* (Fig. 3) as in the whole layer and in the crystal. Consider the outward boundaries of the concentrical belts composed by 6-member cycles. These boundaries can be enumerated by numbers 1, 2, 3, ... n starting from the centre (in the case of Fig. 3, n is equal to 3). In the portions inside the sector the

numbers of carbon atoms are successively equal to 1, 3, 5,..., $(2n-1)$ and their sum is $C=n^2$. The number of hydrogen atoms in the sector MON, which are located only on the periphery, is $H=n$. Hence

$$\frac{C}{H}=n. \qquad (1)$$

As seen from Fig. 3,

$$z=2n-1, \qquad (2)$$

and, taking into consideration the actual dimensions of the cycles, we obtain for the diameter of a crystallite (in Å)

$$d_a=4.25\,\frac{C}{H}-1.42, \qquad (3)$$

where $\frac{C}{H}$ is to be determined by chemical analysis. Thus, for benzene for which $C=6$, $H=6$, the formulae (1)—(3) give in accordance with experiment $z=1$ and $d_a=2.83$ Å.

Let us apply these formulae to different coals not subjected to further treatment, for which analytical and X-ray data are available (the contents of oxygen is here not considered).

Kind of coal	% C	% H	z	d_a, Å calc.	d_a, Å obs.	d_c, Å obs.
Irish peat coke(6)	90.1	1.8	7	30	17	10
Dendrite soot from alcohol	93.4	1.3	11	45	—	—
Acetylene soot (7)	95.0	0.8	19	78	21	13
Sugar coal (7)	98.1	0.7	22	94	21	9
Coal from CO (decomposed at 550°) (7)	95.0	0.1	157	668	150	160

Formula (3) gives values for d_a (column 5) of the same order of magnitude as does the experiment (column 6). This shows that the simple assumption on which the above formulae have been derived are approximately true, and the use of formula (3) in our case is justified. For dendrite soot the value of d_a fits into the same series as in the case of non-dendrite coals (where $d_a \geqslant d_c$), viz. between peat coke and acetylene soot.

The fact that the observed values of d_a are higher than the calculated ones, may be taken to mean that less hydrogen is contained in the layer than it was supposed, i. e. that a portion (up to $^3/_4$) of the lateral carbon surface of the crystallite is bare, this process being enhanced by heating and by the activation of coals, when all hydrogen is practically lost (8).

If the alcohol is allowed to pass over the constantan wire in a current of carbon dioxide, other conditions being equal, no soot is formed. This accords well with the theoretical considerations expressed earlier (9). On the contact surface the carbon molecules are apt to decompose into free radicals which are an intermediate stage in the process of soot formation. The rate of oxidation of the radicals by carbon dioxide being higher than the rate of transformation into soot, the latter has no time to be formed, and this is the reason why no soot formation was observed by us in this case. The work is in progress.

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CHEMISTRY

SOLID SOLUTIONS OF CALCIUM AND STRONTIUM ORTHOSILICATES

By N. A. TOROPOV and P. F. KONOVALOV

(Communicated by V. G. Chlopin, Member of the Academy, 9.III.1943)

The continuously increasing use of the salts of orthosilic acid—orthosilicates—in different branches of silicate industry (fire-resistant and ceramic materials, cement) promotes the development of special physical-chemical studies in this domain. Next to materials, the main component of which is the orthosilicate of only one metal, such as forsterite Mg_2SiO_4, dicalcium silicate Ca_2SiO_4 or fayalite Fe_2SiO_4, more complex orthosilicate composition of the type Mg_2SiO_4—Fe_2SiO_4 forsterite and magnesite refractories, Ca_2SiO_4—Ba_2SiO_4 cements and fire-resistant putties etc. attract attention.

The study of two-component systems formed by orthosilicates has been carried out up to now mainly on components of different rock-forming minerals and compounds of metallurgical slags (Mg_2SiO_4, Fe_2SiO_4, $MnSiO_4$, Ca_2SiO_4). Many orthosilicate systems of considerable technical interest were beyond the scope of phase analysis.

In our former paper [1] results were reported on the physical-chemical investigation of a binary orthosilicate system Ca_2SiO_4—Ba_2SiO_4, in which the formation of a continuous series of solid solutions between the components of the system was stated.

A further step in our work was the study of the binary system Ca_2SiO_4—Sr_2SiO_4 the results of which are reported below.

The compositions of this system are distinguished by high refractory properties, and therefore for the synthesis, both of the extremes of the system Ca_2SiO_4 and Sr_2SiO_4 and of the intermediate compositions, the melting of the initial components in the flame of an electric arc was used.

As starting materials chemically pure strontium and calcium carbonates (Kahlbaum) and finely ground rock-crystal were used. The latter, after evaporation with hydrofluoric acid in a platinum crucible, left a residue of 0.07%. The starting materials were carefully mixed in a porcelain mortar and sifted through a sieve with 10 000 openings per 1 cm^2 for better homogenization.

To this mixture a small amount of dextrine and water was added and from the mass obtained cylinders were formed in narrow glass tubes which after drying were brought to melting in the flame of an electric arc.

In the process of melting droplets were formed that fell in a cooling water or mercury bath. Owing to the high crystallization capacity of orthosilicates, the material obtained contained no glossy residue and under the microscope looked completely crystallized. To achieve complete homogenization this material was again crushed, screened and melted anew.

Microscopical control confirmed complete homogeneity of preparations obtained after two melting operations.

The criterion of sufficient homogeneity of the synthetized solid solutions was the absence in the preparations of starting materials, both in the form of carbonates $CaCO_3$ and $SrCO_3$ and in the form of free oxides CaO, SrO and SiO_2.

Besides, in the samples that were studied calcium and strontium metasilicates, that differ from orthosilicates of the same element through lower refraction, were not found.

In systems with solid solutions often in case of insufficient diffusion of the components the formation of zonal crystals is observed. The separate zones, that are usually concentric, differ from each other by their composition and, correspondingly, by their refraction. According to the law of Konovalov the central districts of zonal crystals are richer in the more refractory component.

An example of such a zonal phenomenon in synthetic monticellites of the composition $mCa_2SiO_4 \cdot n(Mg, Fe)_2SiO_4$ has been described by Beliankin, Nicogosian and Fedotiev (2).

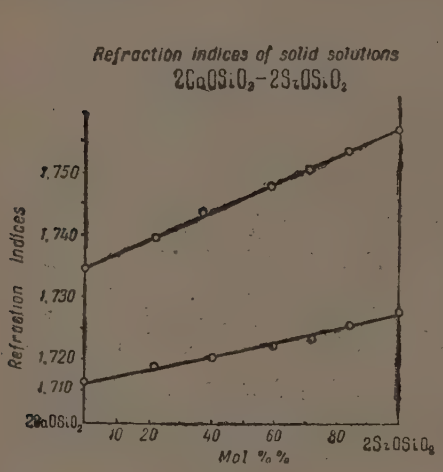

Fig. 1

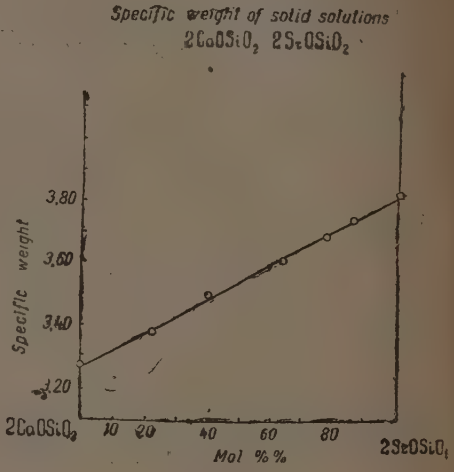

Fig. 2

For all the preparations obtained in the system Ca_2SiO_4—Sr_2SiO_4 and characterized by complete homogeneity and by crystalline structure, the values of the refractive indices (N_g and N_p) were determined in immersion liquids. Measurements were carried out with an accuracy of ± 0.002. The refractive indices of the liquids were controlled on Pulfrich's refractometer. Besides, the specific weights of solid solutions were determined. In order to avoid possible hydration, the determinations of specific weights were carried out using toluene as pycnometric liquid.

The data obtained are given in the table and graphically expressed in Figs. 1 and 2.

Refractive Indices and Specific Weights of Solid Solutions
$2CaO \cdot SiO_2$ — $2SrO \cdot SiO_2$

$2CaO \cdot SiO_2$ % by weight	100	70	50	30	20	10	0
$2SrO \cdot SiO_2$ % by weight	0	30	50	70	80	90	100
$2CaO \cdot SiO_2$, mol.% . .	100	78.0	60.4	39.8	28.08	14.86	0
$2SrO \cdot SiO_2$, mol.% . .	0	22.0	39.4	60.2	71.92	85.14	100
N_gNa	1.735	1.740	1.744	1.747	1.750	1.753	1.756
N_pNa	1.717	1.719	1.720	1.722	1.723	1.725	1.727
d_4^{20}	3.28	3.39	3.513	3.611	3.69	3.74	3.84

An analysis of the graphs makes it possible to state in the system studied the presence of a continuous series of solid solutions, i. e. a complete analogy of this system with the system Ca_2SiO_4—Ba_2SiO_4 studied previously.

According to Eskola ([3]), the series $CaSiO_3$—$SrSiO_3$ is also characterized by the existence of a continuous series of solid solutions, but in the metasilicate series $CaSiO_3$—$BaSiO_3$ we deal with the formation of the chemical compound $2CaO \cdot BaO \cdot 3SiO_2$ that melts with decomposition under the formation of $CaSiO_3$ and a liquid phase at 1320° C. This circumstance points to a greater crystallo-chemical likeness between calcium and strontium compounds compared with calcium and barium, this being also well confirmed by structural relations existing between the aluminates $CaO \cdot Al_2O_3$, $SrO \cdot Al_2O_3$ and $BaO \cdot Al_2O_3$.

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1943. Volume XL, № 4

BIOCHEMISTRY

ON THE METHODS FOR OBTAINING AND ON THE STRUCTURE OF SOME WATER SOLUBLE ANALOGUES OF VITAMIN K

By D. A. BOCHVAR and M. M. SHEMIAKIN

(Communicated by S. S. Nametkin, Member of the Academy, 20.II.1943)

To the serious drawbacks of the natural vitamins K_1 and K_2 and of most of their biologically active synthetic analogues belongs, as well known, their poor solubility in water. Attempts have repeatedly been made by more than one author to obtain highly active analogues of vitamin K soluble in water (1). In the course of our work on compounds of this type we became interested in the observations of Moore (2) concerning the biological properties of the products of reaction between 2-methyl-1, 4-naphthaquinone (vitamin K_3) and bisulphites of alkaline metals. According to Moore's data, 2-methyl-1, 4-naphthaquinone is readily dissolved when heated with solutions of bisulphite. The solutions formed are sufficiently stable and their antihaemorrhagic activity is not below that of 2-methyl-1, 4-naphthaquinone itself. Of particular importance is the fact that these solutions are much less toxic and devoid of the irritating property of the free quinone. On account of the extreme simplicity of obtaining soluble preparations of this type and of their high biological value, Moore's observations deserved serious attention. However, having given an accurate description of the biological properties of the obtained solutions of the products of reaction between 2-methyl-1, 4-naphthaquinone and bisulphites, this author gave no correct interpretation of the processes attending these reactions; neither did he make any attempt to isolate these compounds as individual products. On the ground of indirect evidence Moore believes the dissolving of 2-methyl-1, 4-naphthaquinone in a bisulphite solution to be accompanied by a direct formation of an adduct of type (IV); the latter is supposed to be in aqueous solution in equilibrium with the initial compounds, on the one hand, and with the corresponding hydroquinone sulphonate of type (V), on the other.

We undertook a detailed investigation of the chemical properties of the reaction products of 1, 4-naphthaquinones with bisulphites, both in aqueous solutions and in crystalline state, and we found their behaviour to be in disagreement with what has been said of it by Moore. Neither can those compounds be assigned the structure of normal bisulphite compounds, which might be supposed by analogy with other carbonyl compounds.

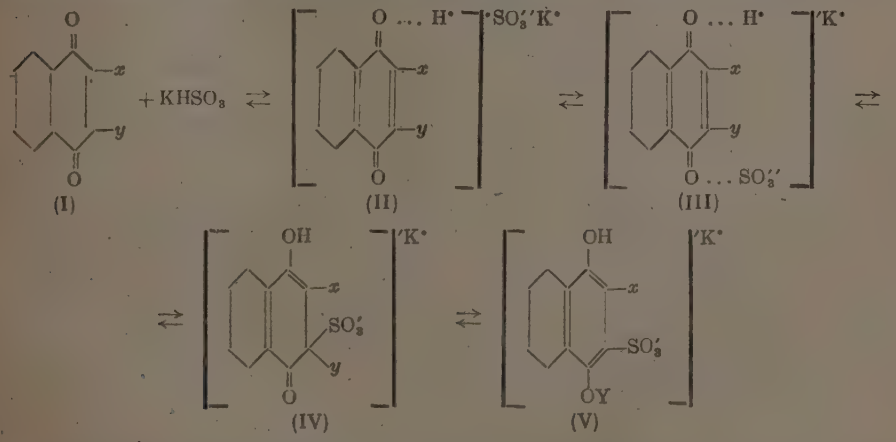
We investigated the products of reaction of bisulphites with 2-methyl-1, 4-naphthaquinone and 2-methyl-1, 4-naphthaquinone-3-sulphonate of potassium; they have been isolated in a crystalline state and contained 1 molecule of bisulphite per 1 molecule of quinone. The following properties have been found to be the most typical of these compounds.

1. They are capable of being oxidized (hydrogen peroxide in acid medium), adding per 1 molecule 1 atom of oxygen. The products of oxidation can be isolated in a crystalline state and contain per 1 molecule of quinone 1 molecule of bisulphate.

2. Certain quinones can supplant other quinones from their compounds with bisulphite. For instance, from the aqueous solution of the product of reaction of bisulphite with 2-methyl-1, 4-naphthaquinone-3-sulphonate of potassium, the latter compound can be displaced by 1, 4-naphthaquinone-2-sulphonate of potassium, 2-methyl-1, 4-naphthaquinone, *p*-benzoquinone, toluquinone. And *vice versa*, from an aqueous solution of the product of reaction of bisulphite with 2-methyl-1, 4-naphthaquinone this quinone can be supplanted by 2-methyl-1, 4-naphthaquinone-3-sulphonate of potassium, though in much smaller amounts.

3. Although compounds of bisulphite with quinones are colourless when in crystalline state, when dissolved in water and especially upon heating of aqueous solutions, there appears a coloration peculiar of the free quinone. The addition of bisulphite in excess reduces the intensity of the colour (e. g., in the case of 2-methyl-1, 4-naphthaquinone-3-sulphonate of potassium). Sometimes the coloration of these aqueous solutions is more intense than that of aqueous solutions of the corresponding free quinones (e. g., of 2-methyl-1, 4-naphthaquinone).

The reactions described above make us reject for compounds of bisulphite with 1, 4-naphthaquinones both the structure of normal bisulphite compounds and the structure expressed by the formula (IV) proposed by Moore. In both cases an equilibrium in aqueous solutions between compounds of this structure and the initial components seems hardly probable, and still less probable appears the possibility of their oxidation as associated with an addition of an atom of oxygen. At the same time, all these properties, as well as a number of other considerations are in favour of the view that the following complex equilibrium is established when 1, 4-naphthaquinones (I) react with solutions of bisulphites of alkaline metals:



where x and y are substituting groups or atoms of hydrogen.

In the case of 2, 3-disubstituted-naphthaquinones (such as 2-methyl-1, 4-naphthaquinone-3-sulphonate) in aqueous solutions there is established equilibrium between the initial components, the corresponding oxonium salt (II) and the bisulphite complex of type (III)*. Unlike this, in case position 2 or 3 is not substituted in the quinone molecule, the compounds are simultaneously in equilibrium with further isomerization products: the adduct of the general type (IV) and the final hydroquinone sulphonate of type (V). The reciprocal transformations (III) and (IV) must be considered as anionotropic processes, while (IV) and (V) must be regarded as cationotropic ones. When the products of reaction of quinones with bisulpnites are isolated in the crystalline state, in the former case there is always formed, irrespective of the conditions of isolation, only the bisulphite complex (III).

* The bisulphite complex (III) is to be considered a resonance of structures, under participation of the corresponding structure with covalent bonds.

In the latter case, when position 2 or 3 is not substituted and the solution shows a more complex equilibrium, for the isolation of the bisulphite complex (III) it is necessary to keep definite temperature conditions, the state of equilibrium being greatly affected, besides the structural factors, also by external factors, especially by temperature. Most frequently a rise in temperature leads to increased yields of the final hydroquinone sulphonate (V). Thus, in the case of a reaction between 2-methyl-1, 4-naphthaquinone and bisulphite, at 30—40° C, there is but very little 2-methyl-1, 4-naphthahydroquinone-3-sulphonate in solution, whereas when heated at 80—120° about 2 hours the yield is up to 60%. The hydroquinone obtained is most easily separated in the form of the corresponding quinone by oxidizing it with potassium bichromate in the presence of sulphuric acid or, best of all, with chlorine. There separates thereby 2-methyl-1, 4-naphthaquinone-3-sulphonate of potassium, sparingly soluble in water; its yield, when oxidized with chlorine, is up to 57%, calculated on 2-methyl-1, 4-naphthaquinone.

As a result of a study of the properties and transformations of products of reaction of 1, 4-naphthaquinones with bisulphites there have been worked out practical methods for preparing highly active water soluble analogues of vitamin K: 2-methyl-1, 4-naphthaquinone-3-sulphonate of potassium and the bisulphite complex of 2-methyl-1, 4-naphthaquinone. Among these particular attention is deserved by the latter, owing to its properties that have already been pointed out by Moore. It should be noted that for use in medical practice it is expedient to prepare the crystalline bisulphite complex of 2-methyl-1, 4-naphthaquinone as a mixture with bisulphite rather than in a pure state, taking 2 moles of the latter for the reaction. This is expedient, since an excess of bisulphite preserves the complex when stored from gradual changes, due to the influence of moisture and oxygen of the air.

Detailed results will be published elsewhere.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
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BIOCHEMISTRY

VARIATION OF ACTIVITY OF TRYPTASE UPON AUTOLYSIS OF MUSCULAR TISSUE

By I. A. SMORODINTZEV and N. V. NIKOLAEVA

(Communicated by S. S. Nametkin, Member of the Academy, 10.II.1943)

Contrary to the prevailing opinion we were able to prove that industrial ripening of meat is attended by no decomposition of proteins which make up the basic part of the meat (1). The reason for the stabilization of meat proteins appears to lie in the fact that the activity of the main proteinase of the tissue, the catepsine, is, according to our experiments (2), diminished by as much as 40—60% in 120 hours after the death of the animal, though the pH remains optimal for catepsine. According to literary data (3), tryptase enters into the composition of the muscular tissue. By tryptase is meant a group of proteinases that act upon the anions of native proteins in a weakly alkaline medium and are activated by enterokinase; sulphohydryl derivatives and hydrocyanic acid have no influence on their activity.

We undertook to trace the variation of the activity of tryptase during the first hours after the killing of the animal, as the meat was being stored under industrial conditions (+3°) or at the temperature of the body (37°).

Method of Investigation. The experiments were run with sound animals, all of a similar type. These were Symmental cows ranging from 6 to 8 years in age. One of the hind quarters was stored in the cooling room at +3°; the other in a thermostat with electric fan at +37°. The chemical tests were checked by microbiological observation.

The meat was freed from fat and connective tissue; carefully minced and extracted with 3 its volumes of water at 37° C during 18 hours in the presence of toluol. The extract was activated for half an hour by a drop of enterokinase. 10 mg of extract were mixed with 20 ml of water in which 0.3 of kasein were suspended; 2 ml of 0.5% soda solution were then added, and the mixture allowed to stand for 6 hours at 37° C under toluol; the pH of the mixture was kept at 8.0—8.2. The degree of splitting was determined from the aminonitrogen by means of a van-Slyke apparatus. At the same time control-tests were carried on containing the enzyme with water and the substratum with water. The results are given in mg % of aminonitrogen (Table 1).

Table 1

First Set of Tests (Averages for a series of animals)

Time elapsed after killing in hours	1	6	12	24	48	72	120	240
				Autolysis at + 3° C				
Aminonitrogen in mg%	60.5	103	99.5	91.5	34.7	46.1	35.1	39.6
Aminonitrogen in % to the first hour	100	170	165	151	57	76	58	65

Continued

Time elapsed after killing in hours	1	6	12	24	48	72	120	240
Autolysis at 37° C								
Aminonitrogen in mg%	60.5	163.4	160	139	41.6			
Aminonitrogen in % to the first hour	100	270	264	230	69			

Table 1 shows that the activity of the tryptase of muscular tissue attains its maximum on the 6th hour after the death of the animal and continues at this high level during 24 hours; then an abrupt decrease by as much as 3 times takes place, and this level is maintained during 10 days of observation at 3°C. The storage temperature has no influence, except that on the 6th hour the activity of tryptase increases nearly 3 times instead of 2, but then the drop of activity toward the 48th hour is not down to $^1/_3$, but to $^1/_4$.

Table 2

Second Set of Tests (Averages for a series of animals)

Time elapsed after killing in hours	1	6	12	24	48	72	120	240
Autolysis at + 3°C								
Aminonitrogen in mg%	4.3	19.4	22	21	13	14	18	18
Aminonitrogen in % to the first hour . .	100	450	510	500	300	326	446	446
Autolysis at + 37°C								
Aminonitrogen in mg%	4.3	52.8	68.2	50.3	18.4			
Aminonitrogen in % to the first hour . .	100	1230	1600	1170	430			

In the second series of tests made on the meat of similar animals the initial activity of tryptase was found to be 15 times as weak as in the animals of the first series, but the dynamics of its variation was in the same direction: the activity of tryptase increased towards the 6th hour after the death of the animal still more considerably (4 to 5 times), and its decrease on the 48th hour was also greater, though down to a higher level than in the first series. In these animals the activity of tryptase is a maximum not on the 6th, but on the 12th hour. At + 37° C the variation of activity is much stronger in these animals than in those of the first set.

Although the activity of the muscular tryptase proper with respect to a foreign protein (kasein) shows a strong increase during the first 24 hours with optimal pH, yet no influence on the muscular proteins can possibly be exercised by this tryptase, because the hydrogen ion concentration is increased in the process of meat ripening.

The intensification of the digestion of meat by a pancreatine added from outside, which, according to our previous observation, takes place on the 6th hour, and the increase of aminonitrogen toward the same time, appear to be partly due to an increase of activity of muscular tissue tryptase at this period.

Thus, our research has shown that the tissue tryptase holds a prominent place in the process of meat ripening, and that the presence of a critical point in the value of different indices after 24 hours (1) is to a great measure accounted for by the state of this enzyme on the autolysis of muscles.

Conclusions

1. The activity of the muscular tissue tryptase proper is liable to considerable variation in animals of the same type.

2. On the autolysis of muscles the activity of tryptase changes in the opposite direction compared to the activity of catepsine.

3. The activity of tryptase attains its maximum on the 6th hour after the killing of the animal and continues at this high level until 24 hours have elapsed.

4. After 48 hours the activity of tryptase shows a decrease by 3—4 times.

5. The decrease of the activity of tryptase after 24 hours is one of the causes of the stabilization of proteins on meat ripening.

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OCEANOLOGY

DIURNAL VERTICAL MIGRATION OF ZOOPLANKTON IN ARCTIC SEAS

By V. G. BOGOROV

(Communicated by P. P. Shirshov, Member of the Academy. 11.III.1943)

The diurnal vertical migration of the plankton is a phenomenon universally observed. Most of the organisms descend during day-time to the deeper layers, often even as deep as to the bottom layers. By night these organisms ascend to the sea surface. Irrespective of the depth, all water layers are periodically either overfilled with organisms, or excessively poor in them. Studies by Waterman, Nunnemacher, Chace, Clarke ([5], [6], [12]) have shown that diurnal vertical migration of plankton proceeds even at a depth of 1000 m.

This phenomenon in the plankton life is due to diurnal variations in the illumination of the sea. The rôle played by light in the control of vertical migration is quite obvious ([5], [6], [9–11]). Attempts to give the phenomenon a different interpretation on the basis of laboratory work are also available ([7], [8]). The artificiality of laboratory tests is notorious. Negligible changes in the chemical properties of the medium are sufficient to make an organism respond in two parallel tests in quite a different manner to light of a similar intensity. It is therefore highly desirable to investigate the phenomenon in such seas where light of a different intensity may be obtained under natural conditions. Such conditions are available in Arctic seas. During summer the observer can work there under conditions of 24-hour illumination; during winter it is a 24-hour darkness. In autumn and spring he has at his disposal alternation of day and night. The polar conditions present a kind of natural laboratory.

At present we dispose of a number of special diurnal stations taken during the Arctic summer in the Barentz and White Seas, and during polar autumn, in the Barentz Sea.

In so far as the results of observations made at different diurnal stations proved very similar, and for lack of space we give here data obtained at two of the stations only. The data are tabulated below.

The table gives a clear picture of how the same organisms behave during summer and autumn as their light conditions change. As a result we are brought to the following conclusions.

1. During the polar summer when the surface of the sea is all the time illuminated, the animals remain always in the same layer. It would seem that under these conditions they are scarcely subject to any diurnal vertical migration. This is, however, in contradiction to the fact of vertical displacement of water particles, brought about by tidal phenomena which in the Barentz Sea([3]) take place throughout tens of meters. Consequently, in order to maintain vertical distribution the organisms have to ascend twice a day when water particles are sinking, and to descend when they rise to the surface. These organisms actively keep at the depth which offers them optimal illumination. Living in one and the same layer during a 24-hour day, they are subject to extreme variations in the surrounding medium. In some of our 24-hour stations the temperature of water showed a di-

Diurnal Vertical Distribution of Mass Species in the Barentz Sea During Summer (Station 133) and Autumn (Station 4687)*

		Calanus finmarchicus						Copepodite stages**					
		I		II		III		IV		V		VI	
		night	day	night	day	night	day	night	day	night	day	night	day
Summer	50—0 m	77	77	75	81	79	85	95	97	79	81	(44)	(28)
	160—50 m	23	23	25	19	21	15	5	3	21	19	(56)	(72)
Autumn	50—0 m							63	4	77	1	92	—
	200—50 m							37	96	23	99	8	100

		Pseudocalanus elongatus						Copepodite stages			
		I+II		III		IV		V		VI	
		night	day	night	day	night	day	night	day	night	day
Summer	50—0 m	80	86	60	70	55	50	68	59	42	32
	160—50 m	20	14	40	30	45	50	32	41	58	68
Autumn	50—0 m					39	10	45	6	73	21
	200—50 m					61	90	55	94	27	79

		Microcalanus pygmeus		*Metridia longa*		*Acartia longiremis*		*Oithona similis*		*Sagitt elegans*		*Oikopleura*	
		night	day	night	day	night	day	night	day	night	day	night	day
Summer	50—0 m	26	19	1	3	100	99	81	98	99	99	58	44
	160—50 m	75	81	99	97	—	1	19	11	1	1	42	56
Autumn	50—0 m	78	26	36	—	70	20	86	80	*Metridia* 95	2	*Oithona plumifera* 23	40
	200—50 m	22	75	64	100	30	80	14	20	5	98	77	60

* The number of organisms is given in per cent. Samples taken from separate runs are classed into: a) superficial layer 50—0 m and b) benthal layer 200—50 m. Those taken at different time of 24-hour day are arranged into «night» (from 18 to 4—6 o'clock) and «day» series (from 6 to 18 o'clock).

** Summer samples contain very few *Calanus*. The data for *Calanus*, stage VI, may therefore be insignificant and taken into parentheses.

urnal change within up to 3° C. In spite of this, the organisms persistently kept at a definite level, although for most of them such temperature variations touch at the limit of their geographical distribution.

Such is the specific type of diurnal vertical migration of the plankton in Arctic seas under the conditions of 24-hour illumination ([1], [2]).

2. During polar autumn, when there is alternation of day and night, the same species begin actively to change their level of habitation in the course of 24 hours. Most of them stay at day-time in the deep layers to rise at the approach of night towards the sea surface; they thus perform diurnal vertical migration, which is typical of more southerly seas. Indirect observations carried on in other Arctic seas ([4]) (not at special stations) showed that in autumn in the Strait of Wilkizky it was during night that the superficial layer was especially rich in organisms. According to my own observations made during 1934 in the south-western part of the Kara Sea, the biomass of the upper layer plankton made up 200 mg during night, whereas at day-time it was 115 mg.

There are but two species, viz. *Oithona similis* and *Oithona plumifera*, which do not change their habitat along the vertical, but those species do not migrate in other seas either.

Thus, the same species which do not change in the course of a 24-hour day the level of their habitat in summer, when there is 24-hour illumination, starts moving actively along the vertical as soon as at the approach of autumn there sets in alternation of day and night. These observations go to confirm the decisive, leading part of light in the phenomenon of diurnal vertical migration. Living at a constant depth throughout a 24-hour day, or changing the level of their habitat, the organisms live all the time under highly variable conditions with respect to temperature, salinity, food, etc. It should be noted that in order either to maintain their habitation level constant, or to change it, the organisms are bound to swim striving not only against the water layer of a given level, but also against the water particles which ascend and sink as a result of tidal phenomena.

3. All these data lead us to a conclusion that the ecological characteristics of a given species cannot be determined unless its diurnal migration is duly taken into account. All water layers which are traversed by the migrating organisms present conditions that are typical of the existence of those animals. The diurnal migration of the plankton is by no means a casual or unusual phenomenon, but a normal life activity of planktonic organisms, reoccurring every day and every hour.

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PHYTOPATHOLOGY

PROOF OF INFECTIOUSNESS OF A PURIFIED PROTEIN PREPARATION OF OAT MOSAIC VIRUS (ZAKUKLIVANIE IN OATS)

By K. S. SUKHOV

(Communicated by L. A. Orbeli, Member of the Academy, 27.IV.1943)

The present work completes the cycle of this author's investigations devoted to intracellular inclusions that develop in oats affected with mosaic.

In 1937 the author discovered numerous needle-like crystals in the cells of a mosaic oat (2) and in 1938 he observed, along with these crystals, the presence of loop-like inclusions (3). Both kinds of inclusion turned out to be specific for the disease in question, and were not detected in sound plants. Preliminary microchemical reactions testified to their protein nature, and the author expressed the opinion that the substance of these inclusions is the mosaic virus.

In 1938 Sukhov and Vovk established the carrier of oat mosaic; this was *Delphax striatella* Fallen. Sukhov and Sukhova found further (1939) that the virus of oat mosaic goes through a long incubation period in the organism of the insect before the latter becomes able to transmit the disease (4). In the same year Sukhov found that sterile insects, fed upon an affected oat, stored in their intestine large amounts of needle-like or prismatic crystals which are absent in the intestine of insects fed on sound oats (5). In histological preparations the crystals from the intestine of the insects and from the affected cells of the oats responded similarly to the same dyes, and the author put forward the assumption in favour of their identity.

In 1940 the author investigated the internal organs of the virus carrying insects and found them to contain specific inclusions in the cells of the intestine epithelium of the fat body and of the salivary glands; these inclusions he described by the name of *X*-bodies. The *X*-bodies were never present in the organism of sterile insects. Though amorphous, they showed a specific affinity to the same dyes as characterize the crystals from the intestine or from the cells of affected oats. They showed special histotropism with respect to salivary glands, in whose cells large accumulations of the inclusion substance were observed (6).

From these observations the author came to conclude that in the oat mosaic we have fortunately a case when it is possible morphologically to detect the virus not merely in the vegetable organism, but also in the organism of the insect carrier, with which this virus is associated organically.

In order to obtain a rigorous proof of the virus nature of these inclusions, it was essential to prepare a purified preparation of the crystals and to secure evidence of their infectiousness.

By a method developed by himself the author in the summer of 1941 obtained a purified protein preparation of winter-wheat mosaic virus, which is similar to the virus of oat mosaic (7). In the autumn of 1941 the author together with Vovk and Alexeyeva applied the same method in order to obtain a purified

crystalline preparation of the oat mosaic virus. Here by means of a biuretic reaction the protein nature of the crystals was proved for the first time in a rigorous way.

It remained to prove the infectiousness of the preparation. The oat mosaic, as has been shown by Sukhov and Vovk (2), is not transmitted artificially by the juice, but rather requires the participation of *Delphax striatella* as a carrier (3).

Therefore for proving the infectiousness of the preparation we had to infect the insects with it. This experiment was run on the experimental field of the Ufa Field-Crop Station. The first tests were made on insects collected in the field, regarding which it was not known whether they were carriers of the disease or not.

Nymphs of the third instar were chosen for the test. As the source of infection served a dense suspension of a purified preparation of the virus. The preparation was 7 months old. The crystalline suspension was placed in a thin glass tube, the lower end of which was sealed up by a thin wax film 10—15 μ thick. A small amount of sucrose was added to the suspension, and the lower end of the tube introduced into a celophane bag containing the insects. The bag had its free edge bound tightly about the glass tube by means of threads.

It was not long after the experiment was started that many of the insects passed on to the wax film, and began to suck the solution. Two days after the insects were transferred under an insulator on to sound oat seedlings. By that time a considerable portion of them (30—50%) had perished, but those which had not, remained alive after being transferred to the oats.

After feeding on the crystalline suspension, part of the insects were dissected. In 7 out of 10 the intestine was found to be filled with crystals, presenting the same picture which we observed in the insects fed upon diseased oats (5).

The experiment was started on May 16; and on June 12 the first mosaic plants appeared under the insulator. In all, 30 insects were placed under the insulator. They provoked the disease in eleven plants (out of 17), i. e. 64% were affected. Under the control insulator 300 insects were put; they were caught in the field simultaneously with the test insects. By them only two plants, in 92, were affected i. e. about 2%. Obviously, there was a considerable difference between the experiment and the control. Yet, the affection with mosaic under the control insulator showed that the insects in the field had not been sterile. This was later confirmed by field observations which proved that oats in the Ufa district were that year infested with mosaic to an extent of 1—2%. Therefore we repeated the experiment with sterile insects introduced for the purpose under insulators.

The experiment was started on June 18 on 60 insects. On July 17 the first mosaic plant appeared under the insulator. On July 19 the affection with mosaic amounted to 18% (17 diseased plants in 94). Under the control insulator, which contained 400 sterile insects, no diseased plants were observed at all, and on the open neighbouring oat plantations the affection was only 1%. The ultimate evaluation made on July 25 revealed 21% of mosaic in the test-set.

We have thus obtained an unmistakable evidence that sterile insects fed upon a suspension of virus crystals accumulate these crystals in their intestine, and that the insects become carriers of the disease after the incubation period is over. This was an ultimate proof of the infectiousness of the protein crystals obtained from oats affected with mosaic.

There were also parallel tests with sterile insects on their infection with the extract from fresh mosaic leaves, which also gave positive results. The extract from dry leaves gave no infection.

These tests permitted to formulate certain properties of the oat mosaic virus:

1. The virus passes through the bacterial filter of Seitz which was used by us for its purification.

2. The virus tolerates standing in a fresh water extract from mosaic leaves during a period of no less than 30 minutes (the limit has not been established).

3. In a purified preparation the virus remained active for no less than 7 months.

4. The virus provokes no infection when the insects are fed upon a water extract from dry mosaic leaves.

The results obtained bear out the author's assumption expressed earlier concerning the value and nature of the protein inclusions observed in the case of oat mosaic. They permit also a deep insight into the domain of organic relationship between certain phytopathogenic viruses and their vectors.

Finally, it has here been rigorously proved that the virus responsible for the disease belonging to the group of yellows ([1]) and connected with the incubation period in the insect, is likewise a protein substance, which fact has so far been proved only for typical mosaics transmitted mechanically.

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ENTOMOLOGY

ON THE EVOLUTION OF FLIGHT IN INSECTS

By B. B. ROHDENDORF

(Communicated by A. A. Borissiak, Member of the Academy, 3.I.1943)

Insects belong to those animals, few in number, that are endowed with the faculty of active flight. Their organs of flight, pair wings, governed by a complicated and strong musculature, consist in a thin chitinous membrane, reinforced by a system of thickened veinlets, in other words, by venation. The structure of venation accurately reflects the mechanical properties of the wing. When investigating the various forms of venation in different insects, we gain an idea of the evolution of flight in these animals.

The most primitive are the wings in some Palaeozoic black-beetles, say, in *Platyblatta*; their primitiveness consists in a uniform distribution of the veinlets which are equally numerous both in the front and in the hind half of the wing; both parts are almost symmetrical as to venation. This symmetry testifies to an equal stress experienced by the wing in the process of flight. This mode of flight is a planing, slow one; the most ancient insects used their wings rather as supporting planes that aided them in jumps from elevated objects.

On account of the peculiarities in the movement of the wing, the rate of flight could not possibly increase unless the anterior edge of the wing was reinforced.

This was attained by a shift of the network of the veinlets anteriorly, by their thickening, by modifications of the anterior edge, its straightening, and, finally, by changes in the central and apical part of the wing. This mechanization process may be retraced when studying any flying insect; the original symmetry of the wing was thereby disturbed, and the wing ultimately acquired its peculiar shape and sharply asymmetrical venation system.

The evolution of flight in some groups of insects was essentially controlled by the necessity of supplying the wing with some defensive appliances. As such appeared the ability of plicating the wings along the body, on the back, and ultimately the front wings became transformed into covering organs, the elytra. In this latter case the flight was effected by the hind pair of wings alone, which led to the development of a sharply pronounced heteronomy of the wings; the progressive evolution of the function of flight went on along the way of formation of various longitudinal and transversal folds on the wings, in order to make the large wings plicatile and concealable under the small elytra. This impeded the process of perfection of the wing mechanism. The most primitive stage of the formation of covering elytra may be observed in *Blattoidea*; the next stage in wing evolution is illustrated by various *Orthopteroidea* whose wings bear traces of mechanical flight specialization.

Extreme stages of wing heteronomy are attained in orders *Coleoptera*, *Dermaptera*, *Strepsiptera*, whose front wings are entirely deprived of the flight function, being often reduced in dimensions; the hind wings attain a high mechanical perfection and assume a conspicuously symmetrical structure, owing to an extreme reinforcement of the costal edge and a weakening of the hind edge («costalization»); such are the wings in various beetles.

A different mode of evolution underwent those insects whose front wings have not lost their chief function, that of flight, and were not converted into covering organs.

They preserved their original harmony (such are the *Odonata*, *Neuroptera*, *Mecoptera*, some of the *Trichoptera* and *Lepidoptera*, and a series of less important groups); it has not been disturbed until a more perfect flight technique was worked out.

The perfection consisted in an intimate union of the wing pairs in the process of flight, first by way of synchronization of their strokes, and further on through the formation of special hooks, hooklets, or folds. This process led to a reduction in the size of the hind wings which became only annexes of the front ones (*Hymenoptera*, *Ephemeroptera*, some of the *Homoptera*, *Lepidoptera*). Ultimately it resulted in that a single pair of flight organs was developed, which is essentially identical with the evolution of flight in the above-analysed insects whose front wings are found to be transformed into covering organs. In either case only one pair of wings preserve their flying faculty. This process, the result of which may be called «dipteroidy» or «dipterization», naturally reflected the requirements of aerodynamics; owing to it, a most perfect flying machine was worked out, and as such should be considered the «two-winged» insect which is endowed with a faculty of good flight.

The validity of this conclusion is supported by the wide occurrence of the «dipterization» process which we observe to take place in a vast majority of the youngest and most progressive groups of insects: in the *Diptera*, *Hymenoptera*, *Lepidoptera* and in beetles.

The «dipterization» consisted not only in a mechanical specialization of the wings, but was accompanied by modifications of the whole body which acquired a most perfect stream-line shape, well adapted aerodynamically to a swift flight. Such bodies approach to the shape of a drop or a cigar with a truncate tip.

A good illustration is offered by the butterflies *Sphingidae*, many *Diptera* (e. g. *Metopiinae*), some bees (e. g. *Coelioxys*); all these insects are endowed with a very swift and dirigible flight.

Dipterization is the most important, but by no means the unique direction along which the evolution of the flight function can proceed. There exist many other types of flight in insects, say, that of dragon-flies, or that of the smallest insects, of *Hymenoptera Mimaridae*, *Ptillidae* beetles etc.

A peculiar mode of flight is inherent to diurnal butterflies and to some *Neuroptera*.

All these types of flight are, however, much less developed among insects and, what is especially important, are much excelled by the two-wing flight as to rate and dirigibility.

The literature on the flight of insects is rather abundant (it comprises about 200 items). It gives an idea of the high standing of the experimental technique worked out to investigate different peculiarities of flight, viz. its velocity, the trajectories of the wing movements during flight, number of wing strokes, etc. At the same time the data are rather fragmentary and not always of the same value.

A great variety of insects were studied by mechanicians; there was no critical choice of the material whatever. This is why the results obtained are insufficient and defy comparison. The main reason which prevented the students from correctly retracing the evolution of flight and which is responsible for the fortuitive and fragmentary character of the studies on the mechanics of flight in insects, is the

lack of any evolutionary approach on the part of the students who lacked adequate knowledge of the taxonomic relations between the groups studied by them. The chief part in investigations of the mechanics of flight in insects should belong to the biologist-taxonomist; it is he who has to point out the material and the sequence of its treatment. This is of particular importance if we want to obtain results valuable both from the evolutionary point of view and useful for purely practical purposes.

Besides, being little versed in insect morphology, mechanicians used to overlook the structural details of the insect's body, in particular, those of the wings. And this considerably affected the value and the accuracy of the mechanical analysis of insects, made by them. To give an example, the wings of insects are now known to be a highly complicated mechanical structure which is anything but like to those simple flat membranes as they have usually been thought of. The venation of the wings, various brittles and hairs on them are in as good an agreement with the requirements of aerodynamics, as the rest of the details of the body of a flying insect.

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